EECI-IGSC Course

Networked Model Predictive Control for Multi-Vehicle Decision-Making

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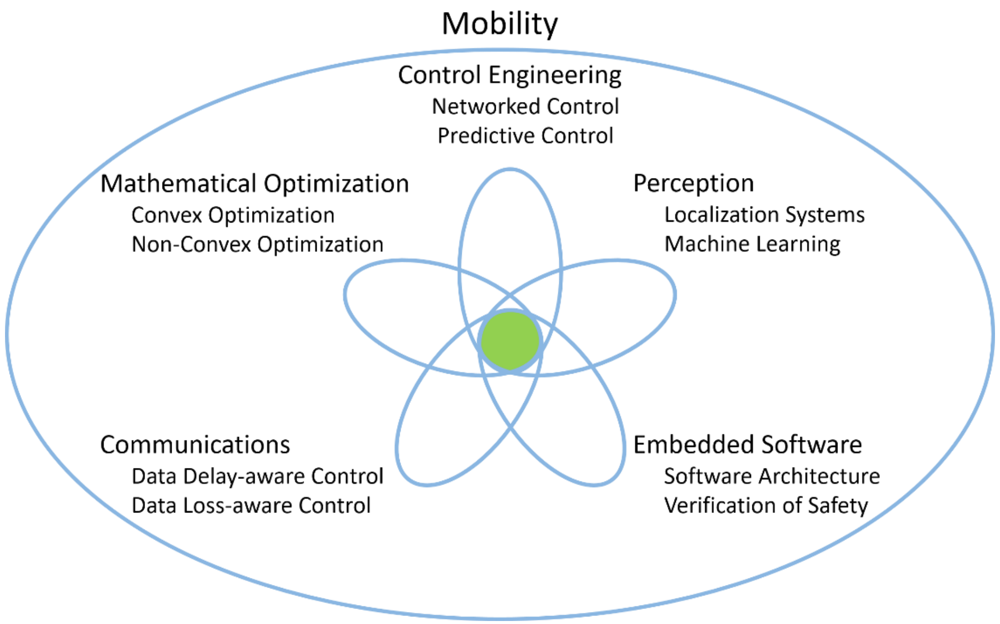
Part 2

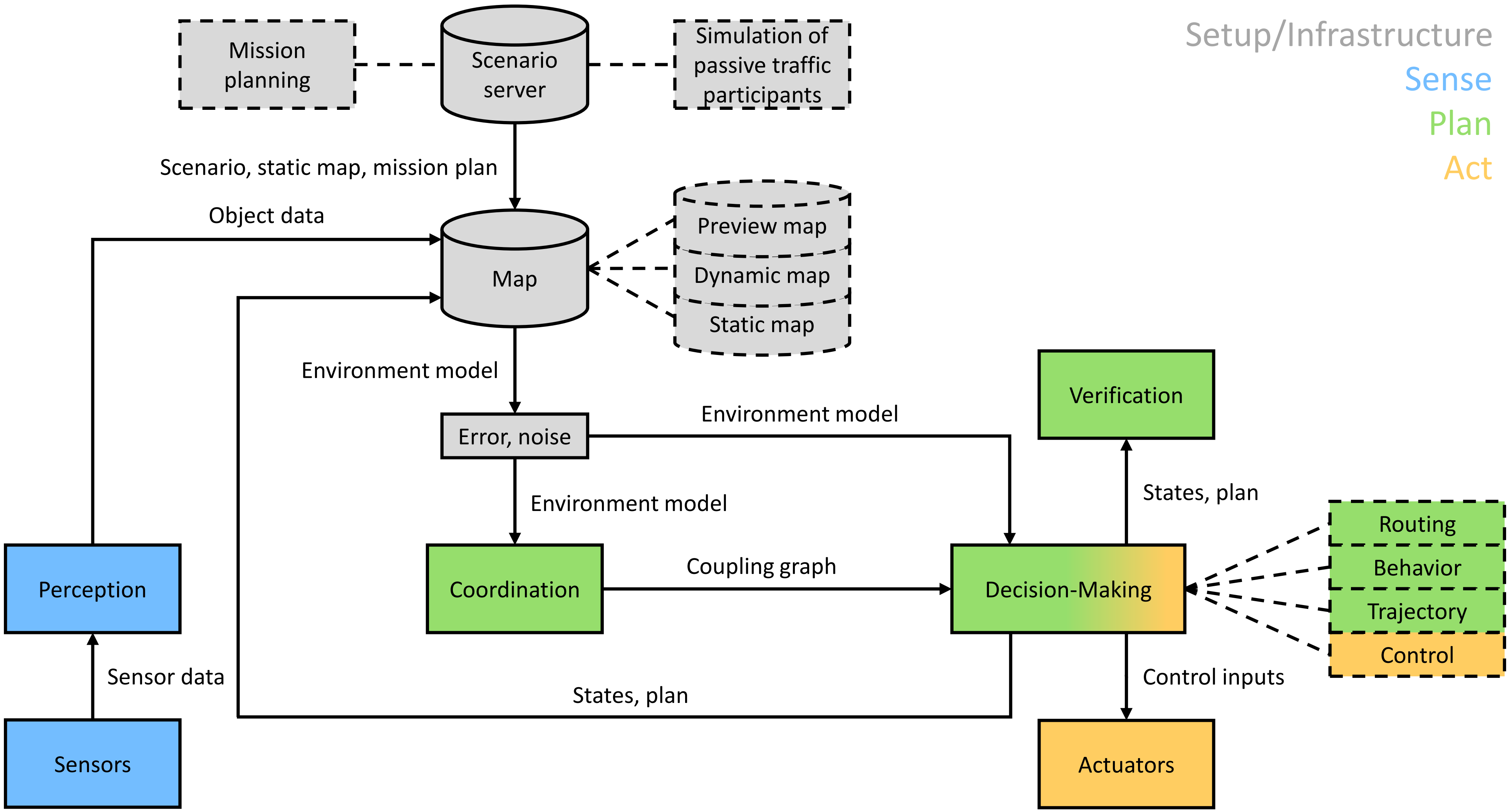
Vehicle Models

Course contents

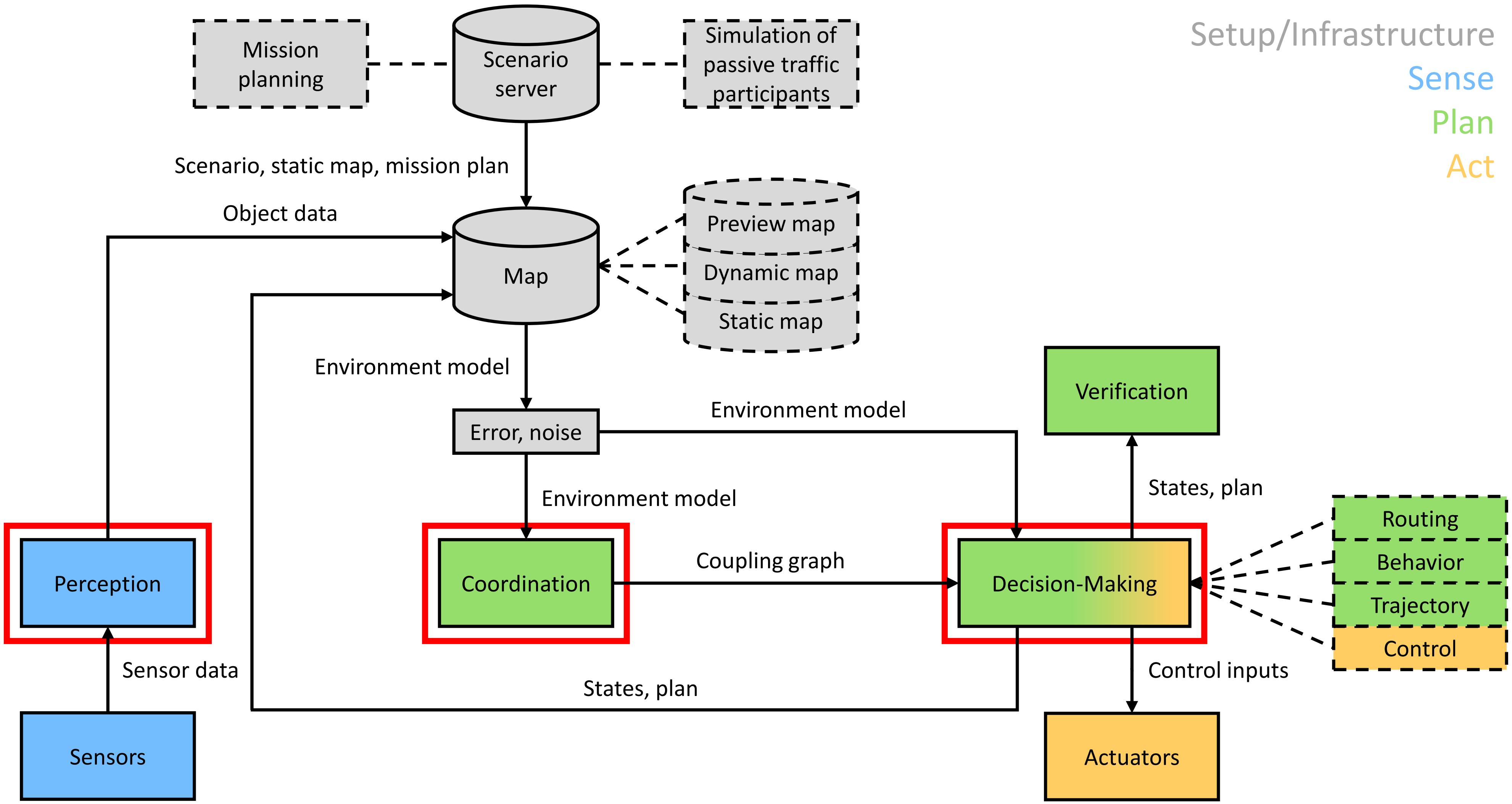
- Vehicle models
- Control and optimization
- Network and distribution
- Software architectures and testing concepts



CPM Lab architecture



CPM Lab architecture



Literature

- R. Rajamani. Vehicle Dynamics and Control. Springer, 2005.

Further literature (1)

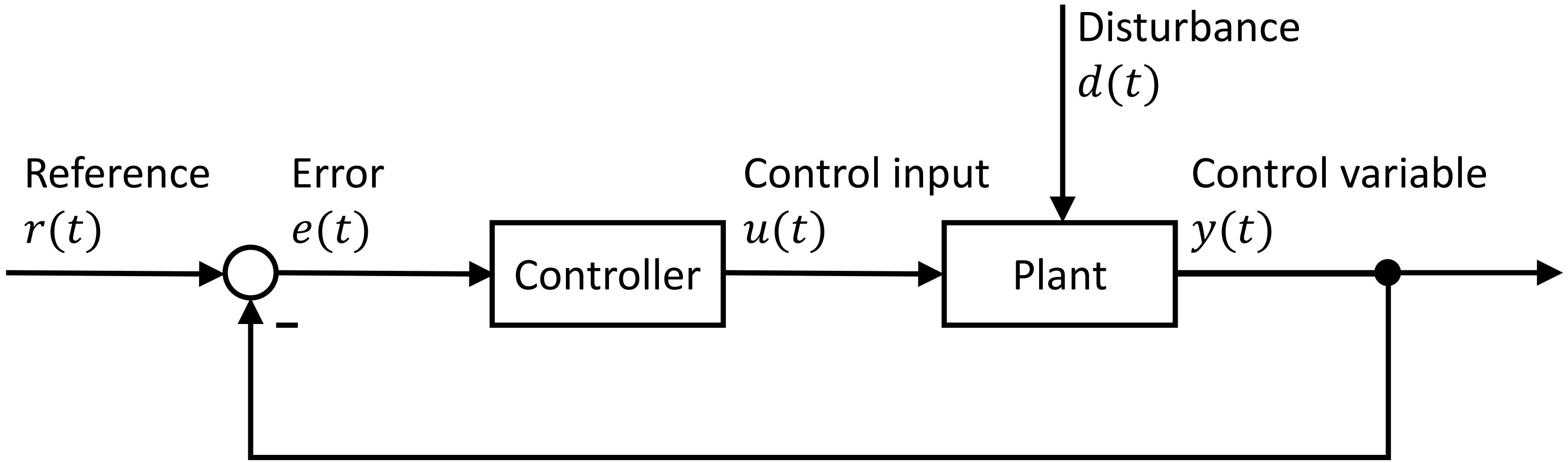
- M. Althoff, M. Koschi, and S. Manzinger. CommonRoad: Composable Benchmarks for Motion Planning on Roads. In IEEE Intelligent Vehicles Symposium, 2017.
- M. Althoff. CommonRoad: Vehicle Models. 2019 link

Further literature (2)

- B. Alrifaee. Networked Model Predictive Control for Vehicle Collision Avoidance. PhD thesis, RWTH Aachen University, 2017.

Control loop

- Elements of control loop (sense, plan, act)
- Example: speed control

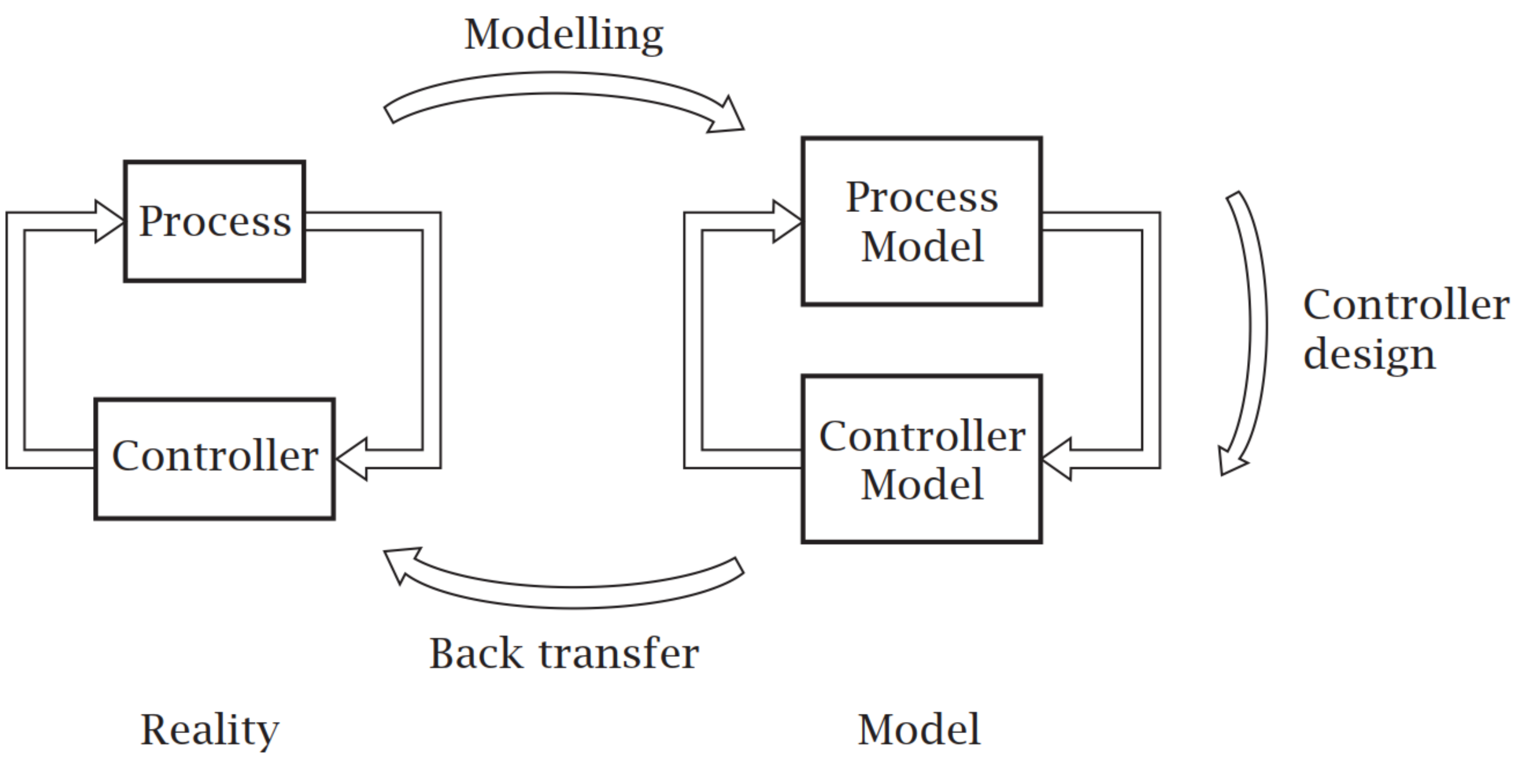


Definitions

- Model
- Simulation
- Model for process simulation
- Model for control

Model

- Model reflects only a part of the characteristics of the original



Abel, Automatic Control, 2009

Modelling steps

1. Modelling goal
2. Model assumptions
3. Verbal description
4. Block diagram
5. Model equations
6. Model validation

Lunze, Regelungstechnik 1, 2016, pp. 42-43

Vehicle models

- Longitudinal model
- Lateral models
  - Point-mass
  - Kinematic single-track*
  - Single-track
  - Multi-body
- Reading (lateral models)
  - R. Rajamani. Vehicle Dynamics and Control. Springer, 2005.
    - Sections 2.2 and 2.3, pages 20-33
  - B. Alrifaee. Networked Model Predictive Control for Vehicle Collision Avoidance. PhD thesis, RWTH Aachen University, 2017.
    - Section 2.4, pages 15-18
  - M. Althoff. CommonRoad: Vehicle Models. 2019 link

*Kinematic ~ geometric, single-track or bicycle

Model validation

- Identification and validation using data from
  - a multi-body model
  - a more detailed simulation software
  - the lab
  - a real vehicle

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General remarks

- Due to clarity of presentation, we will sometimes omit the time argument
- Pay attention to the frames
  - (x, y) is the global coordinate system
  - $(\mathrm{long}, \mathrm{lat})$ is the vehicle's coordinate system

Steering, position, velocity and acceleration constraints

$$\delta \in [\underline{\delta}, \overline{\delta}], \quad v_\delta \in [\underline{v}_\delta, \overline{v}_\delta]$$

$$(s_x, s_y) \in \mathcal{S}_x \times \mathcal{S}_y, \quad v \in [\underline{v}, \overline{v}]$$

$$a_{\texttt{long}} \in [\underline{a}, \overline{a}(v)], \quad \overline{a}(v) = \begin{cases} a_{\max} \frac{v_s}{v} & \text{for } v > v_s \\ a_{\max} & \text{otherwise} \end{cases}$$

$$\dot{a}_{\texttt{long}} \in [\underline{\dot{a}}, \overline{\dot{a}}]$$

- v_s is the speed above which the acceleration is limited by the engine power and no longer by the tire friction

Althoff, 2019

Steering, position, velocity and acceleration constraints

- Friction circle, aka Kamm's circle

$$\sqrt{a_{\mathtt{long}}^2 + a_{\mathtt{lat}}^2} \leq a_{\mathtt{max}}, \quad a_{\mathtt{lat}} \approx v\dot{\Psi}$$

- Kamm's circle is a good approximation since the peak forces of tires are almost identical for the longitudinal and lateral direction

Althoff, 2019

Steering and acceleration constraints

$$v_\delta = f_{\text{steer}}\left(\delta, v_{\delta,d}\right) = \begin{cases} 0 & \text{for } \left(\delta \leq \underline{\delta} \wedge v_{\delta,d} \leq 0\right) \vee \left(\delta \geq \overline{\delta} \wedge v_{\delta,d} \geq 0\right) \quad (C1) \\ \underline{v}_\delta & \text{for } \neg C1 \wedge v_{\delta,d} \leq \underline{v}_\delta \\ \overline{v}_\delta & \text{for } \neg C1 \wedge v_{\delta,d} \geq \overline{v}_\delta \\ v_{\delta,d} & \text{otherwise} \end{cases}$$

$$a_{\mathtt{long}} = f_{acc}\left(v, a_{\mathtt{long,d}}\right) = \begin{cases} 0 & \text{for } \left(v \leq \underline{v} \wedge a_{\mathtt{long,d}} \leq 0\right) \vee \left(v \geq \overline{v} \wedge a_{\mathtt{long,d}} \geq 0\right) \quad (C2) \\ \underline{a} & \text{for } \neg C2 \wedge a_{\mathtt{long},d} \leq \underline{a} \\ \overline{a}(v) & \text{for } \neg C2 \wedge a_{\mathtt{long,d}} \geq \overline{a}(v) \\ a_{\mathtt{long,d}} & \text{otherwise} \end{cases}$$

Althoff, 2019

Steering and acceleration constraints – discussion

- Control input and its limits

  δ or v_δ, note that δ and v_δ limit $a_{\texttt{lat}}$ and $\dot{a}_{\texttt{lat}}$

  $a_{\texttt{long}}$, note that $\dot{a}_{\texttt{long}}$ is for comfort

- Accuracy
  - Wideness (relaxation, over-approximation) vs.
  - Tightness (restriction, under-approximation)

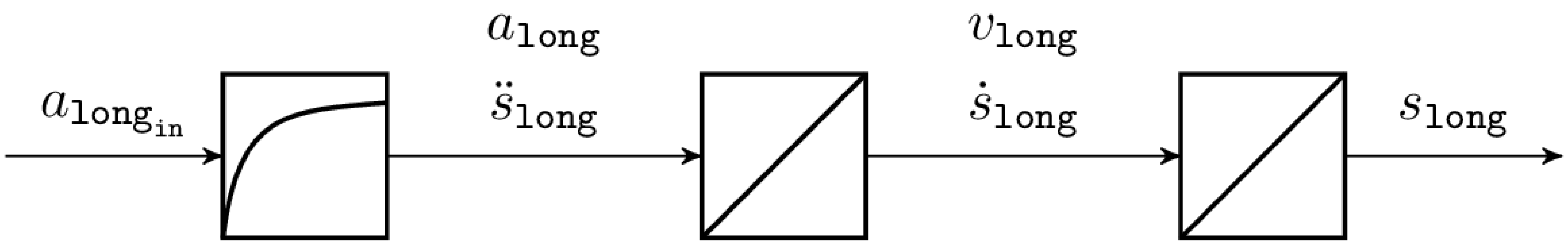
Longitudinal model

- First order differential equation

$$\dot{a}_{\mathtt{long}}(t) = \frac{-1}{T_a} a_{\mathtt{long}}(t) + \frac{1}{T_a} a_{\mathtt{long}_{\mathtt{in}}}(t)$$

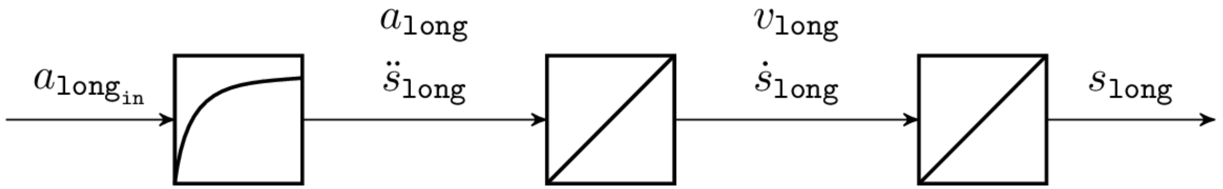
  - Lag (time constant) 0.5-0.7sec

- Block diagram



Longitudinal model

- State space model



$$x_1 = s_{\text{long}}$$
$$x_2 = \dot{s}_{\text{long}} = v_{\text{long}}$$
$$x_3 = \ddot{s}_{\text{long}} = a_{\text{long}}$$
$$u = a_{\text{long}_{\text{in}}}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -\frac{1}{T}x_3 + \frac{1}{T}u$$

$$\dot{x} = Ax + Bu \Leftrightarrow \dot{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T} \end{bmatrix} u$$

Longitudinal model

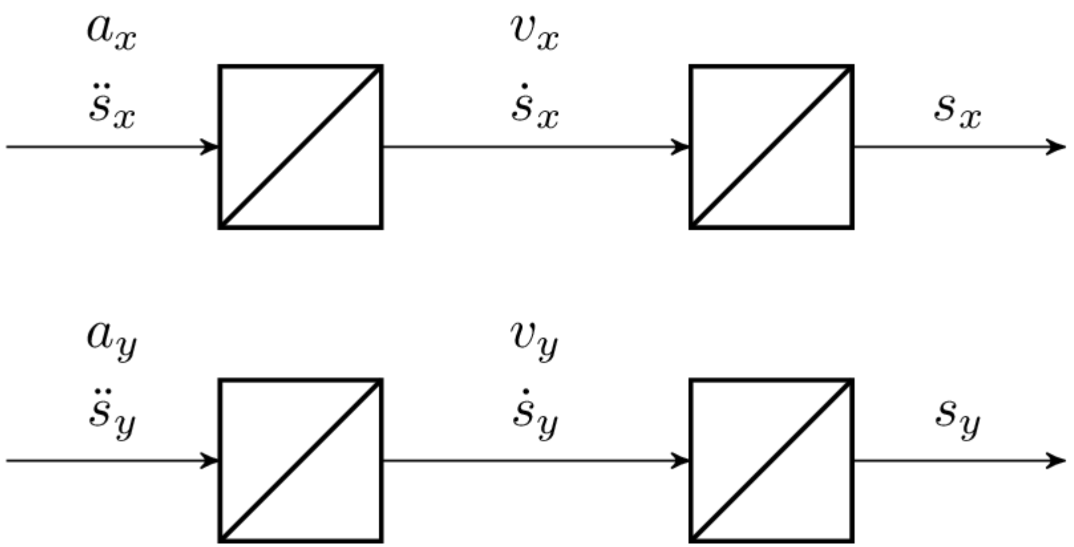
- Useful, e.g., for
  - Speed control, also called Cruise Control (CC)
  - Speed and distance control, also called Adaptive Cruise Control (ACC)
  - Distance control of multiple vehicles
  - Less for stop-and-go
- Discussion on modelling steps

1. Modelling goal
2. Model assumptions
3. Verbal description
4. Block diagram
5. Model equations
6. Model validation

Point-mass model

$$\ddot{s}_x = a_x, \quad \ddot{s}_y = a_y, \quad \sqrt{a_x^2 + a_y^2} \leq a_{\max}$$

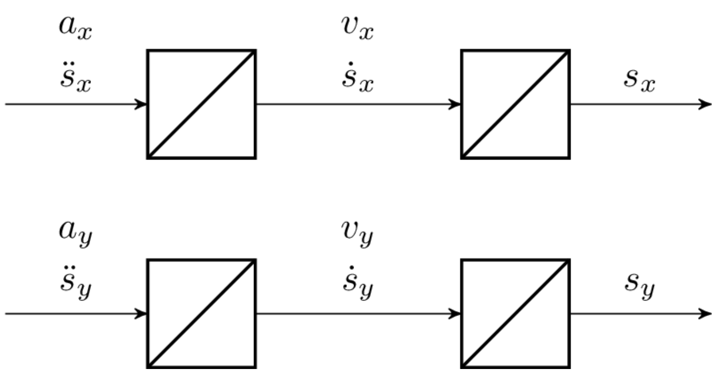
- Ignores minimum turning radius
- Block diagram



Althoff, 2019

Point-mass model

- State space model



$$x_1 = s_x$$
$$x_2 = s_y$$
$$x_3 = \dot{s}_x = v_x$$
$$x_4 = \dot{s}_y = v_y$$
$$u_1 = a_x$$
$$u_2 = a_y$$

$$\dot{x}_1 = x_3$$
$$\dot{x}_2 = x_4$$
$$\dot{x}_3 = u_1$$
$$\dot{x}_4 = u_2$$

$$\dot{x} = Ax + Bu \Leftrightarrow \dot{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Althoff, 2019

Point-mass model

- Usefulness depends on application
- Discussion on modelling steps

1. Modelling goal
2. Model assumptions
3. Verbal description
4. Block diagram
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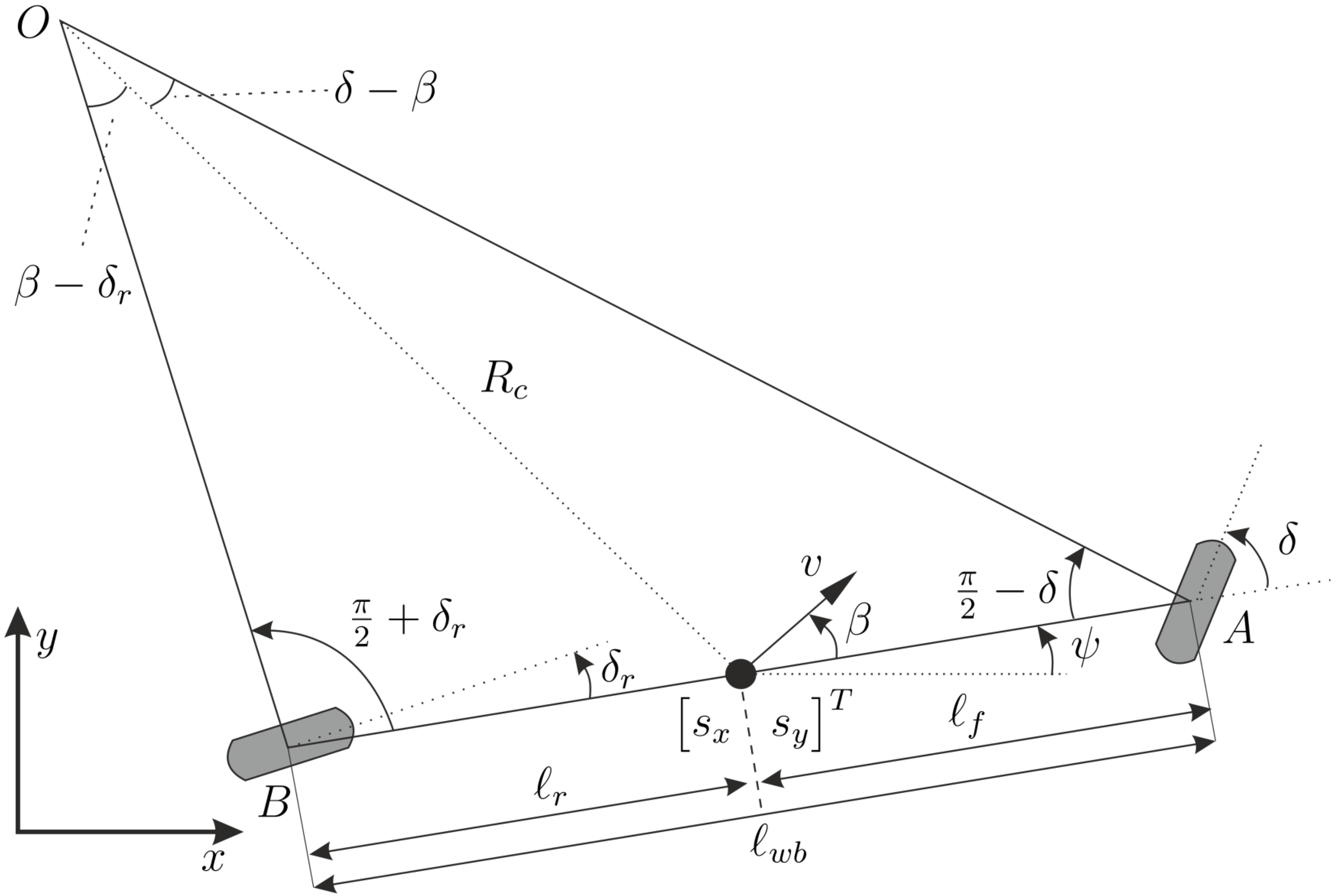
Kinematic single-track model

- Reading
  - R. Rajamani. Vehicle Dynamics and Control. Springer, 2005.
    - Section 2.2, pages 20-27
- Additional reading **[optional]**
  - B. Alrifaee. Networked Model Predictive Control for Vehicle Collision Avoidance. PhD thesis, RWTH Aachen University, 2017.
    - Section 2.4, pages 15-18
  - M. Althoff. CommonRoad: Vehicle Models. 2019 link
    - Chapter 5, pages 4-6

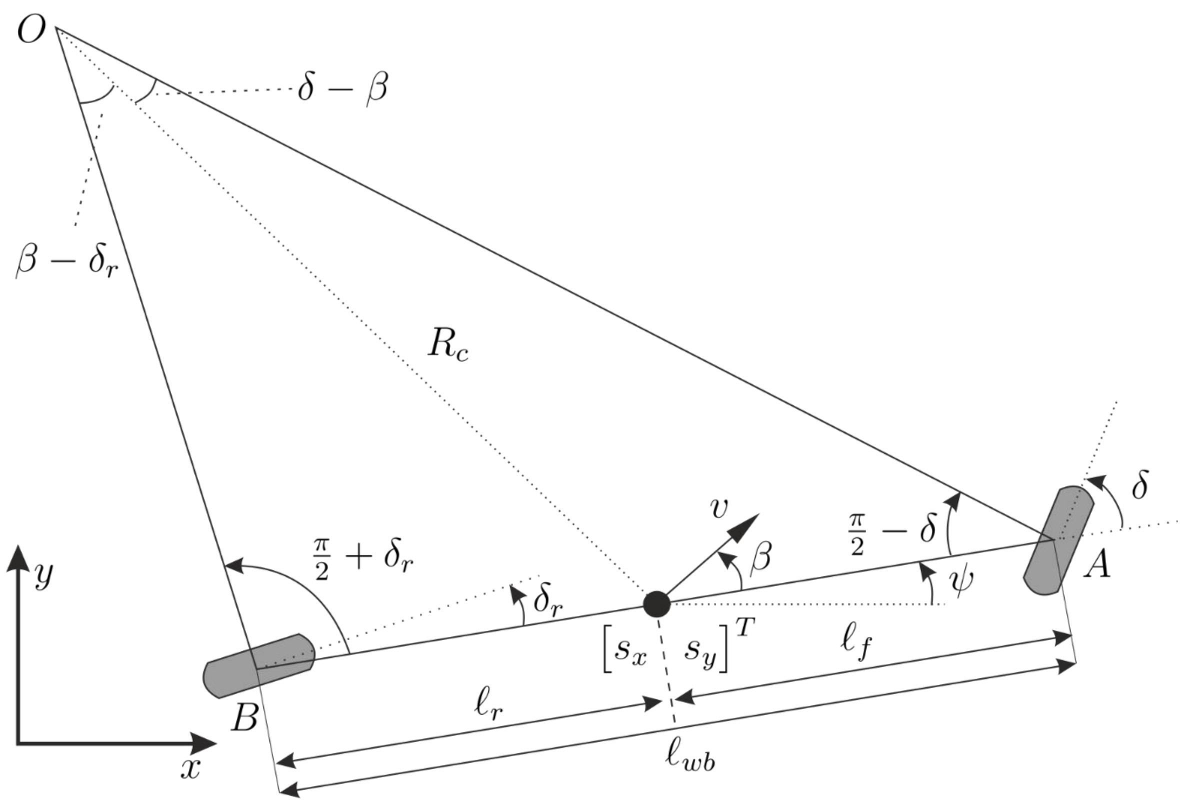
Kinematic single-track model



Alrifaee, 2017

Kinematic model – parameters and variables

Variable/Parameter	Description
$\begin{bmatrix} s_x(t) & s_y(t) \end{bmatrix}^T$	Position of the CG
$\psi(t)$	Yaw* angle
$v(t)$	Vehicle velocity at the CG
$\delta(t)$	Front wheel steering angle
$\delta_{in}(t)$	Input steering angle
$v_\delta(t)$	Velocity of $\delta(t)$
T_δ	Time constant of the steering system
$\beta(t)$	Side slip angle
$\ell_{wb} = \ell_f + \ell_r$	Wheelbase



*or orientation or heading

Kinematic model – assumptions

1. The height of the Center of Gravity (CG) is assumed to be zero, hence the movement of the vehicle is planar, and thus pitch and roll dynamics are neglected

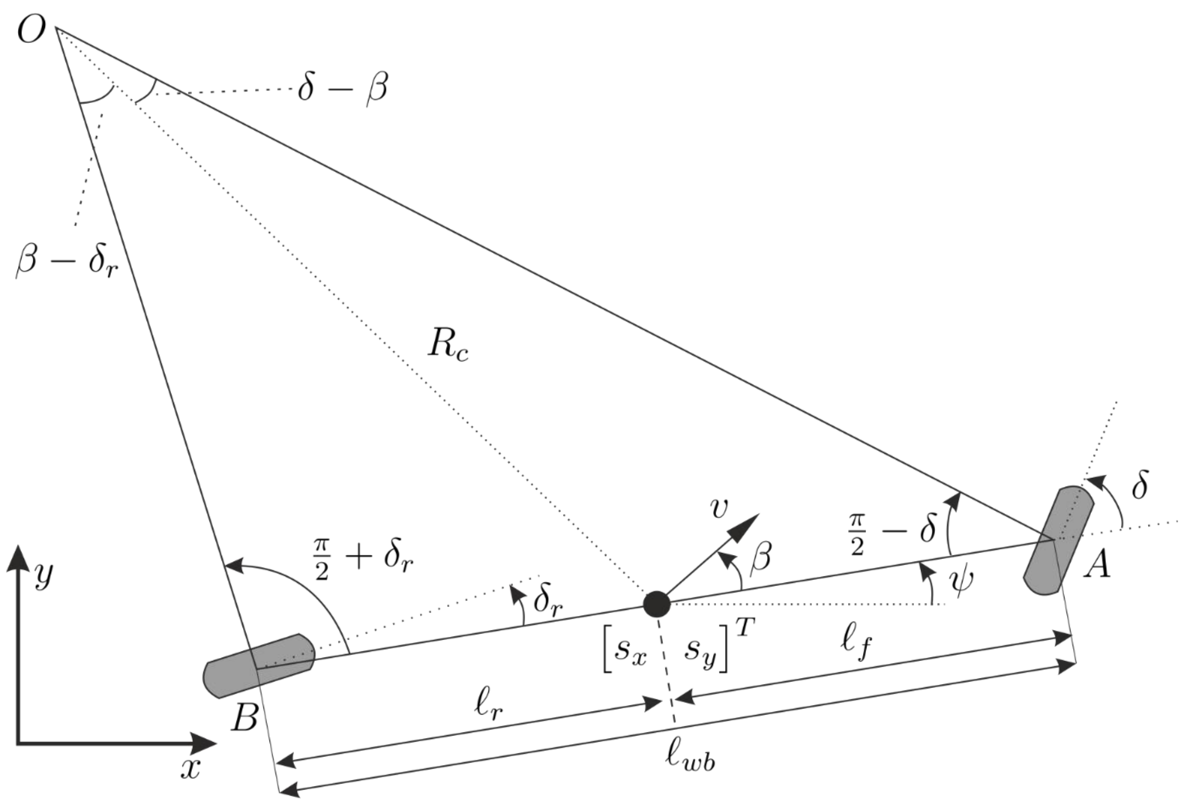
2. The front wheels are represented by one wheel at point A, and the rear wheels are represented by one wheel at point B

3. Rolling resistances and aerodynamic drag are neglected

4. The vehicle is equipped with front-wheel-only steering ($\delta_r = 0$)

5. Forces are neither applied at the front nor at the rear tire

6. The velocity vectors at points A and B are parallel to the front and rear wheels, respectively



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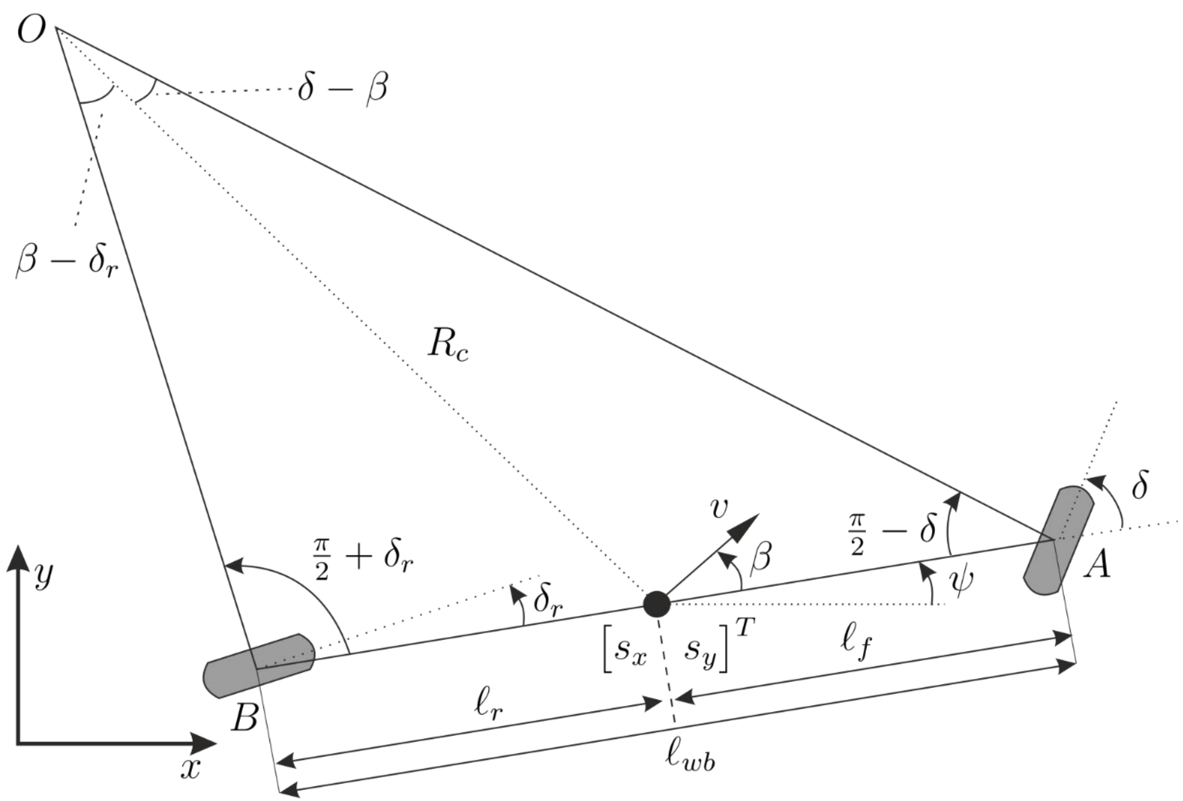

► **Which assumption is strong*? Why?**

*a strong assumption leads to a major difference between model and reality

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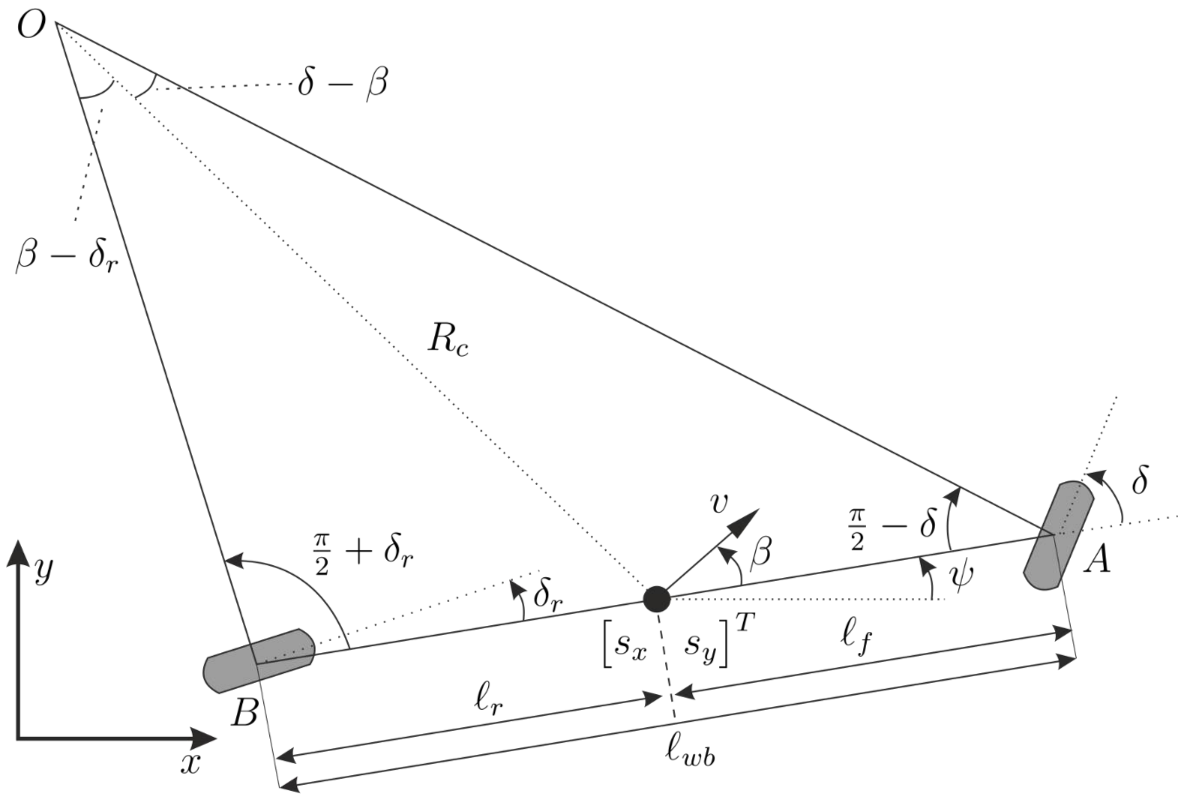
6. The velocity vectors at points A and B are parallel to the front and rear wheels, respectively

   In general valid for small tires' lateral forces, i.e., low speed or big radius

$$f_{\mathtt{lat}} \approx m\frac{v^2}{R_c} \Rightarrow v_{\max} = \sqrt{a_{\mathtt{lat},\max} R_c}$$



Kinematic model – derivation of equations

$$\dot{s}_x(t) = v(t)\cos(\psi(t) + \beta(t))$$

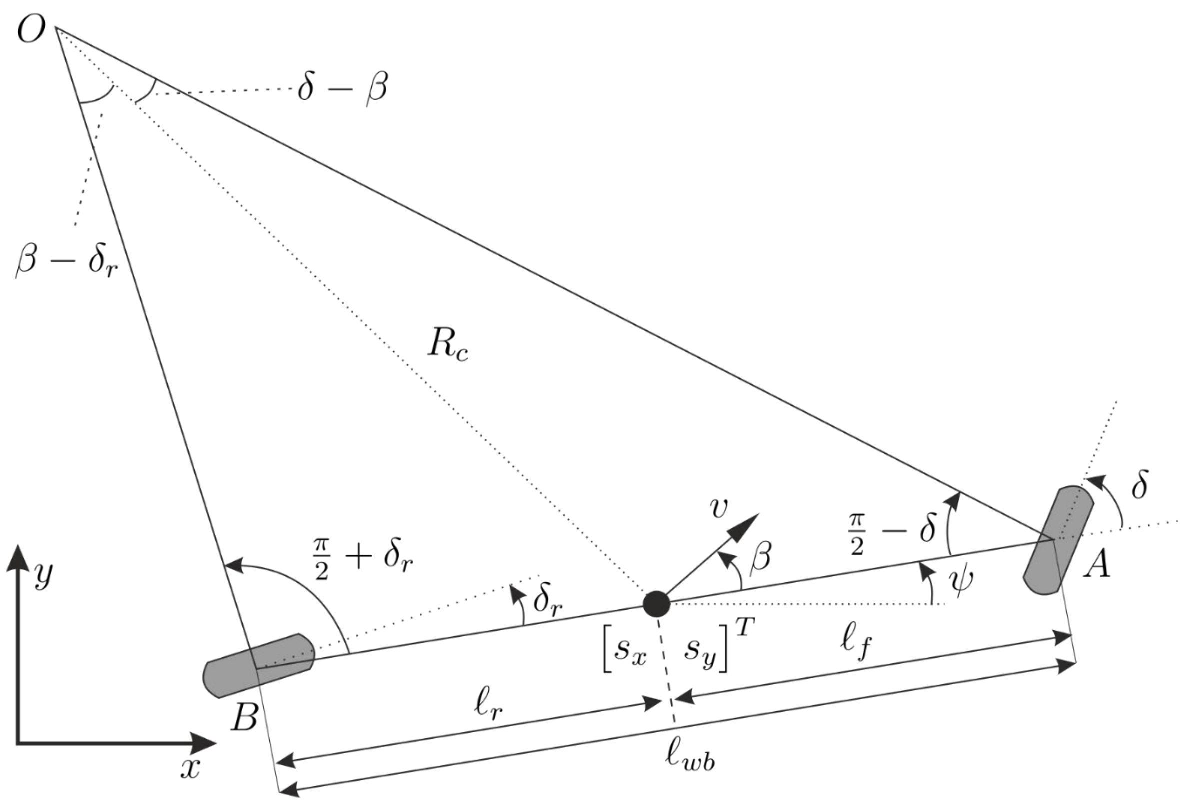
$$\dot{s}_y(t) = v(t)\sin(\psi(t) + \beta(t))$$

$$\dot{\psi}(t) = ?$$

$$\dot{v}(t) = ?$$

$$\dot{\delta}(t) = ?$$

$$\beta(t) = ?$$



Kinematic model – derivation of equations

- At low speeds

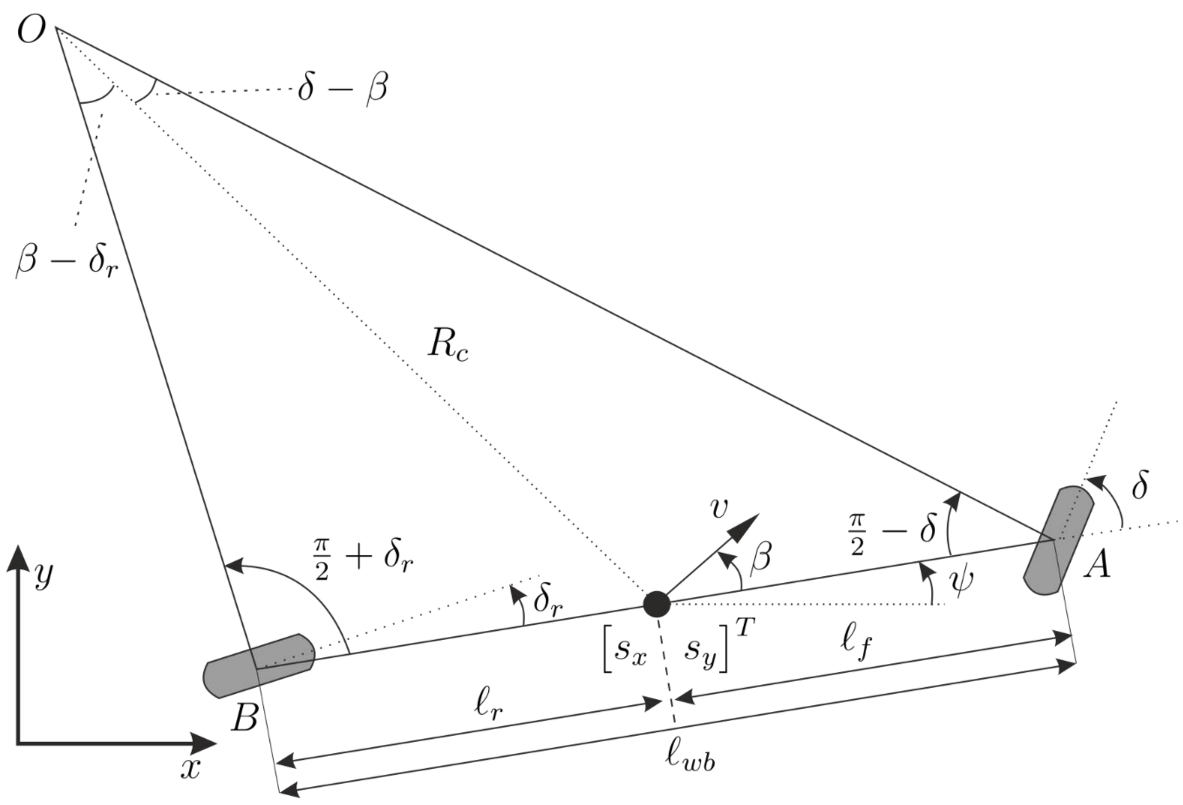
$$\dot{\psi}(t) \approx \frac{v(t)}{R_c}$$

- Applying the sine rule in triangles and some trigonometric functions

$$R_c = \frac{\ell_{wb}}{\tan\delta(t)\cos\beta(t)}$$

$$\dot{\psi}(t) = \frac{1}{\ell_{wb}} v(t) \tan\delta(t) \cos\beta(t)$$

$$\beta(t) = \tan^{-1}\left(\frac{\ell_r}{\ell_{wb}} \tan\delta(t)\right)$$



Kinematic model – equations

- Velocity/acceleration is external input

$$\dot{v}(t) = 0 \text{ or}$$

$$\dot{v}(t) = a(t) \text{ or}$$

$$\dot{v}(t) = a_{\texttt{long}}(t) \text{ if } \beta(t) = 0 \text{ or}$$

$$\dot{v}(t) = \frac{-1}{T_v} v(t) + \frac{1}{T_v} v_{in}(t)$$

Kinematic model – equations

- Steering input

$$\delta(t) \text{ or}$$

$$\delta_{in}(t) \text{ with } \dot{\delta}(t) = \frac{-1}{T_\delta}\delta(t) + \frac{1}{T_\delta}\delta_{in}(t) \text{ or}$$

$$v_\delta(t)$$

Kinematic model – equations

$$\dot{s}_x(t) = v(t)\cos(\psi(t) + \beta(t))$$

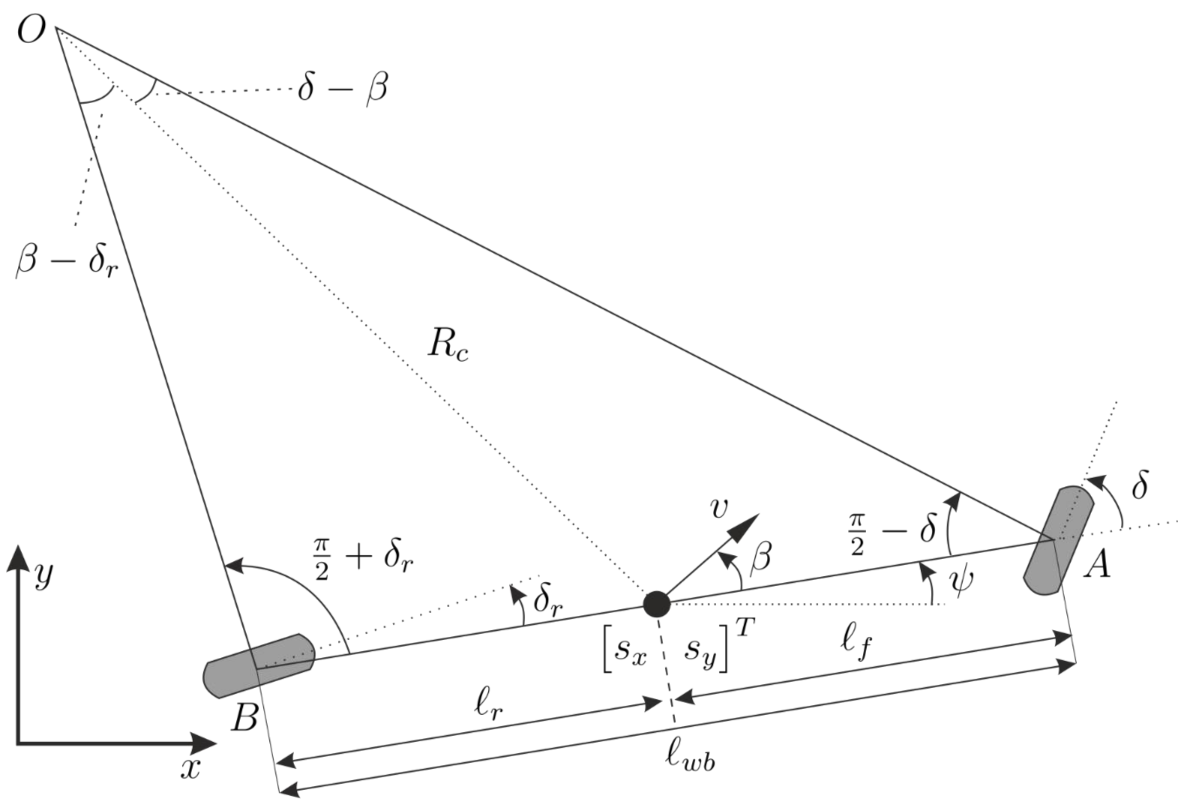
$$\dot{s}_y(t) = v(t)\sin(\psi(t) + \beta(t))$$

$$\dot{\psi}(t) = \frac{1}{\ell_{wb}} v(t) \tan\delta(t) \cos\beta(t)$$

$$\dot{v}(t) = \text{several ways}$$

$$\dot{\delta}(t) = \text{several ways}$$

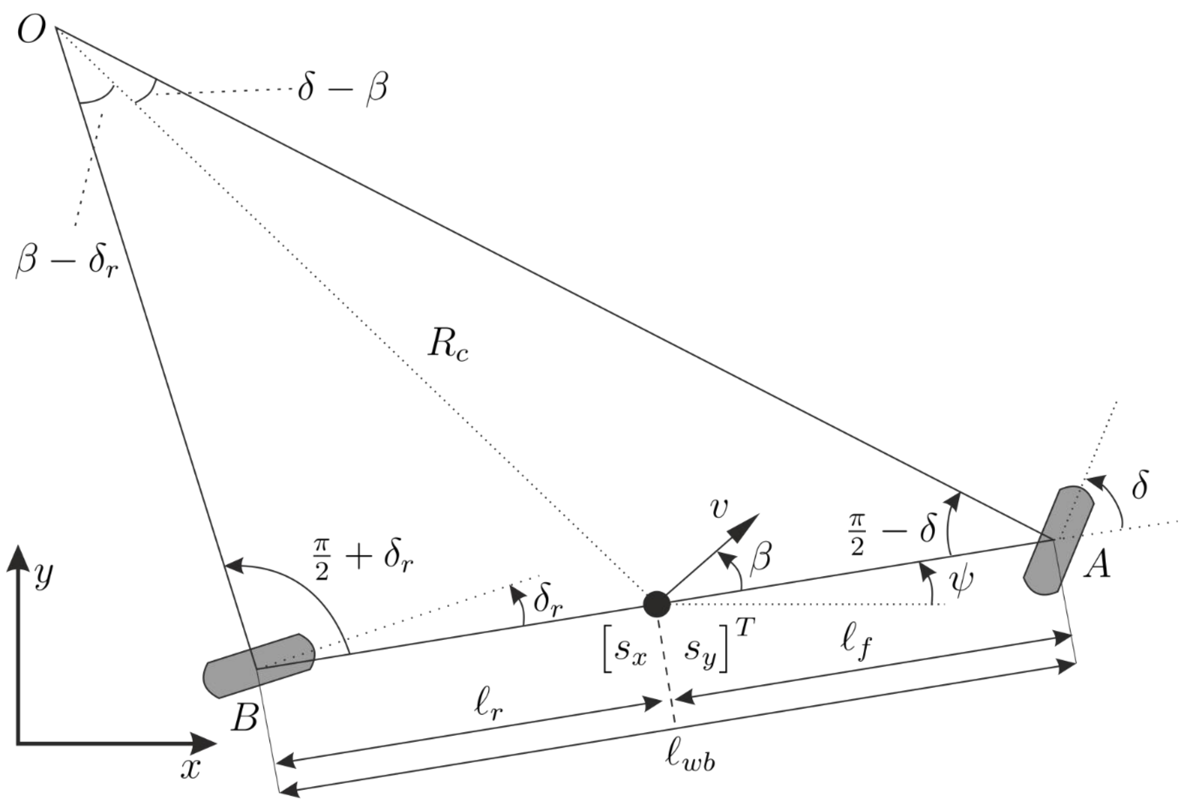
$$\beta(t) = \tan^{-1}\left(\frac{\ell_r}{\ell_{wb}} \tan\delta(t)\right)$$



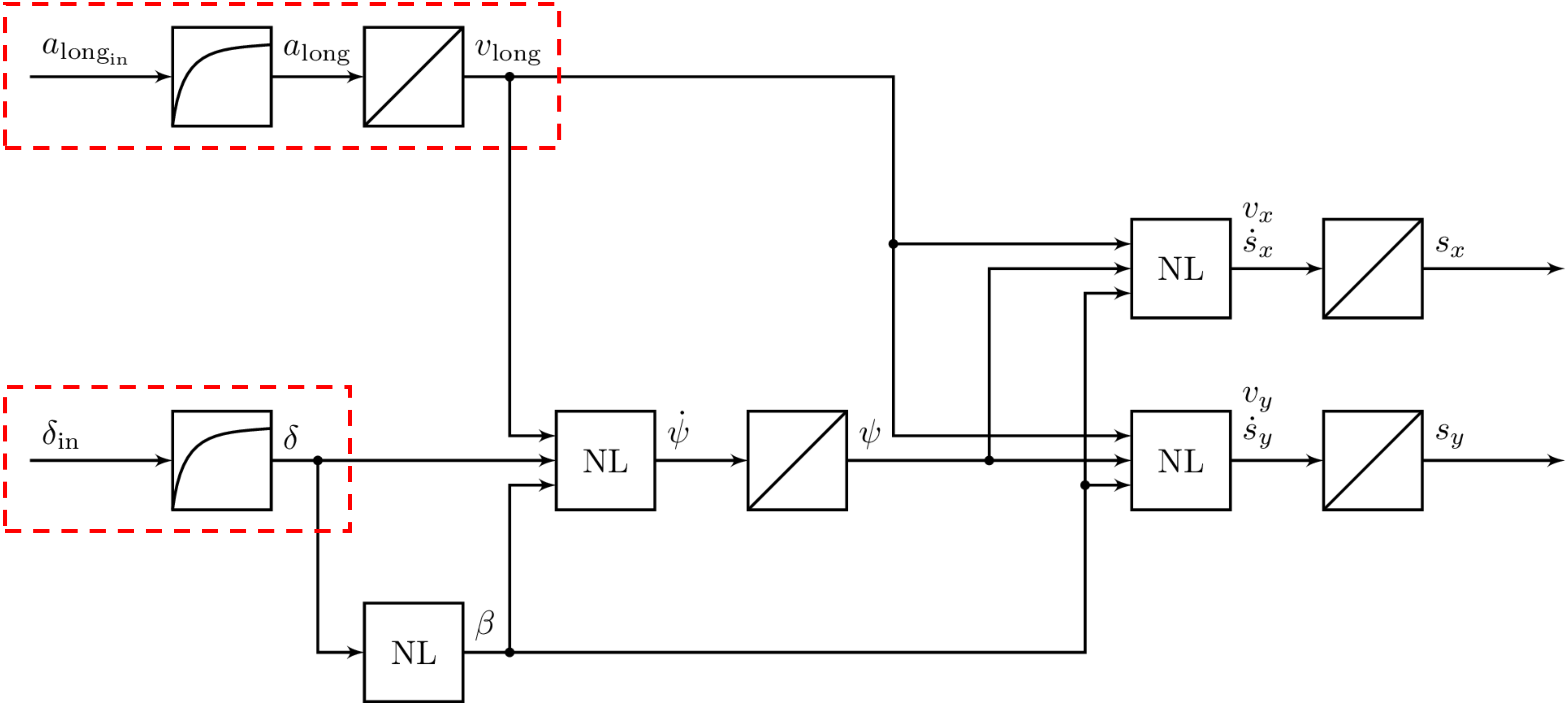
Kinematic model – steering constraint

$$a_{\texttt{lat}}(t) \approx \frac{v(t)^2}{R_c} = \frac{v(t)^2 \tan(\delta(t)) \cos(\beta(t))}{\ell_{wb}}$$

$$\delta_{\max}(t) = \tan^{-1}\left(\frac{a_{\texttt{lat},\max}\ell_{wb}}{v(t)^2 \cos(\beta(t))}\right)$$



Kinematic model – block diagram



Kinematic model – state space model

$$x_1 = s_x \qquad \dot{x}_1 = x_4 \cos\left(x_3 + \tan^{-1}\left(\frac{\ell_r}{\ell_{wb}} \tan x_5\right)\right)$$

$$x_2 = s_y \qquad \dot{x}_2 = x_4 \sin\left(x_3 + \tan^{-1}\left(\frac{\ell_r}{\ell_{wb}} \tan x_5\right)\right)$$

$$x_3 = \psi \qquad \dot{x}_3 = \frac{1}{\ell_{wb}} x_4 \tan x_5 \cos\left(\tan^{-1}\left(\frac{\ell_r}{\ell_{wb}} \tan x_5\right)\right)$$

$$(x_4 = v \qquad \dot{x}_4 = 0 \text{ or } \ldots \text{ or external input without need for } x_4)$$

$$(x_5 = \delta \qquad \dot{x}_5 = u \text{ or } \dot{x}_5 = -\frac{1}{T_\delta} x_5 + \frac{1}{T_\delta} u)$$

$$u = v_\delta \text{ or } \delta_{in} \text{ or } \delta \text{ without need for } x_5$$

Kinematic model

- Usefulness depends on application
- Discussion on modelling steps

1. Modelling goal
2. Model assumptions
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Kinematic model – discussion

- Side slip angle, speed at different points along the wheelbase

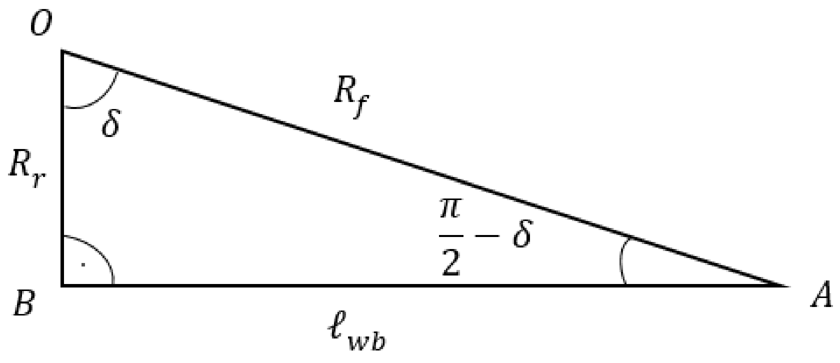
$$v_c = \dot{\psi} R_c$$

$$v_r = \dot{\psi} R_r$$

$$\frac{v_c}{v_r} = \frac{R_c}{R_r}$$

$$R_c = \sqrt{R_r^2 + \ell_r^2}$$

$$\tan\delta = \frac{\ell_{wb}}{R_r}$$



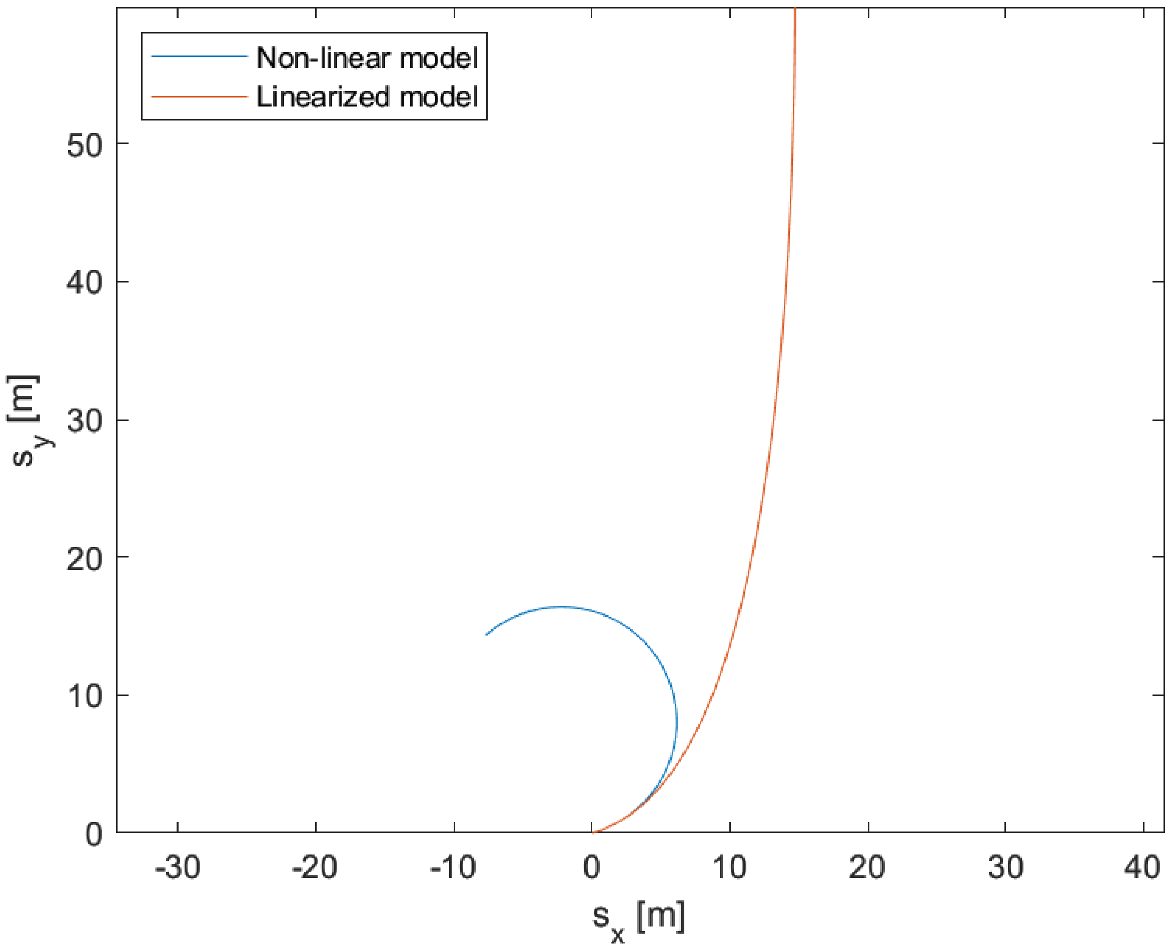
$$R_r = \frac{\ell_{wb}}{\tan\delta}$$

$$v_c = v_r \frac{\sqrt{\left(\frac{L}{\tan\delta}\right)^2 + \ell_r^2}}{\frac{\ell_{wb}}{\tan\delta}}$$

$$v_c = v_r \frac{\sqrt{\ell_{wb}^2 + \ell_r^2 \tan^2\delta}}{\ell_{wb}}$$

Kinematic model – discussion

- Linearization issues



Single-track model

- Cancel assumption
  - 6. The velocity vectors at points A and B are parallel to the front and rear wheels, respectively
- Use
  - Newton’s second law
  - Moment balance

Single-track model

- Cancelling more assumptions leads to
  - More accurate/complex models
  - More difficulties in tracing errors
  - Increasing effort of parametrization
- Usefulness depends on application
  - Example www.sturmkind.com/de/drift-hybrid-gaming

Multi-body model

- Cancel assumptions
- Useful as
  - a reference model
  - a simulation model

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Evaluation of lateral models

- Use
  - https://gitlab.lrz.de/tum-cps/commonroad-vehicle-models/tree/master/MATLAB

Next Part

Control engineering and optimization

- Model predictive control
- Sequential convex programming