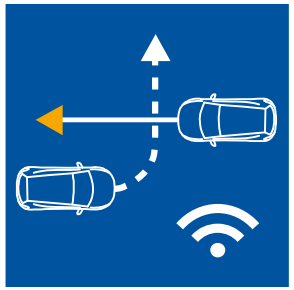


EECI-IGSC Course

Networked Model Predictive Control for Multi-Vehicle Decision-Making

Dr.-Ing. Bassam Alrifaee | Patrick Scheffe, M. Sc.

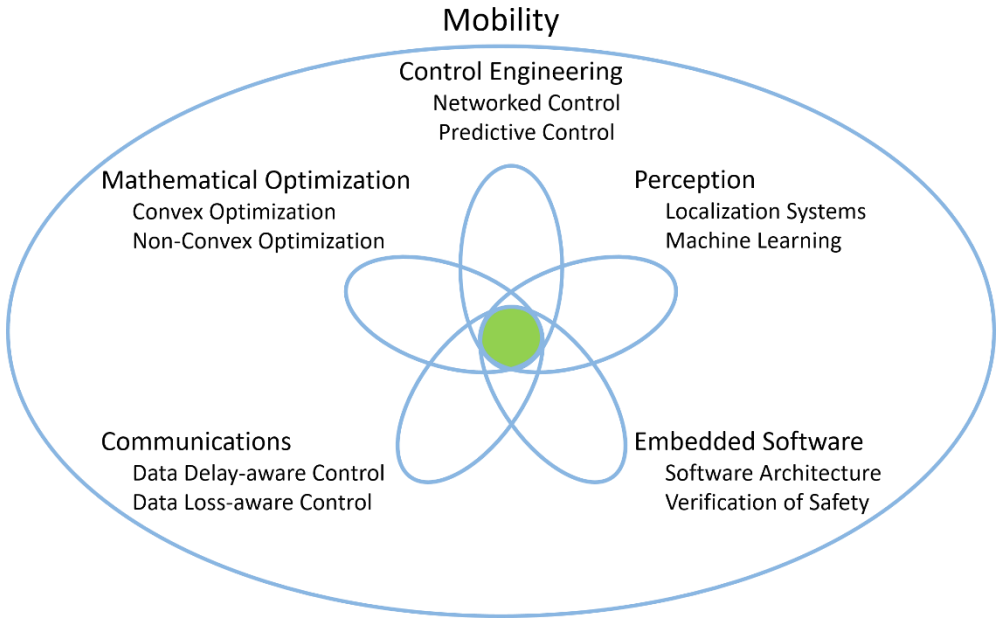
2021

Part 4

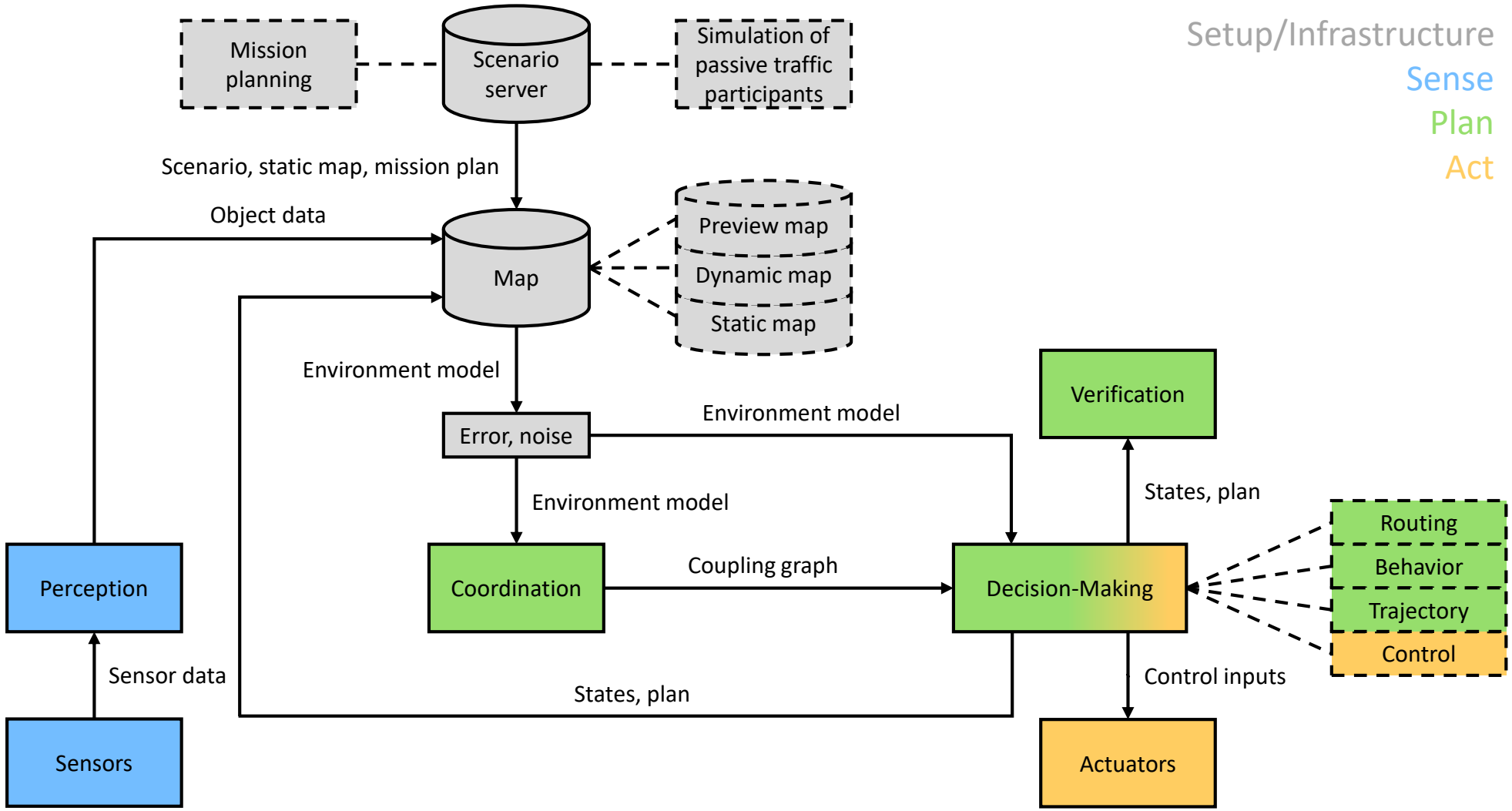
Network and Distribution

Course contents

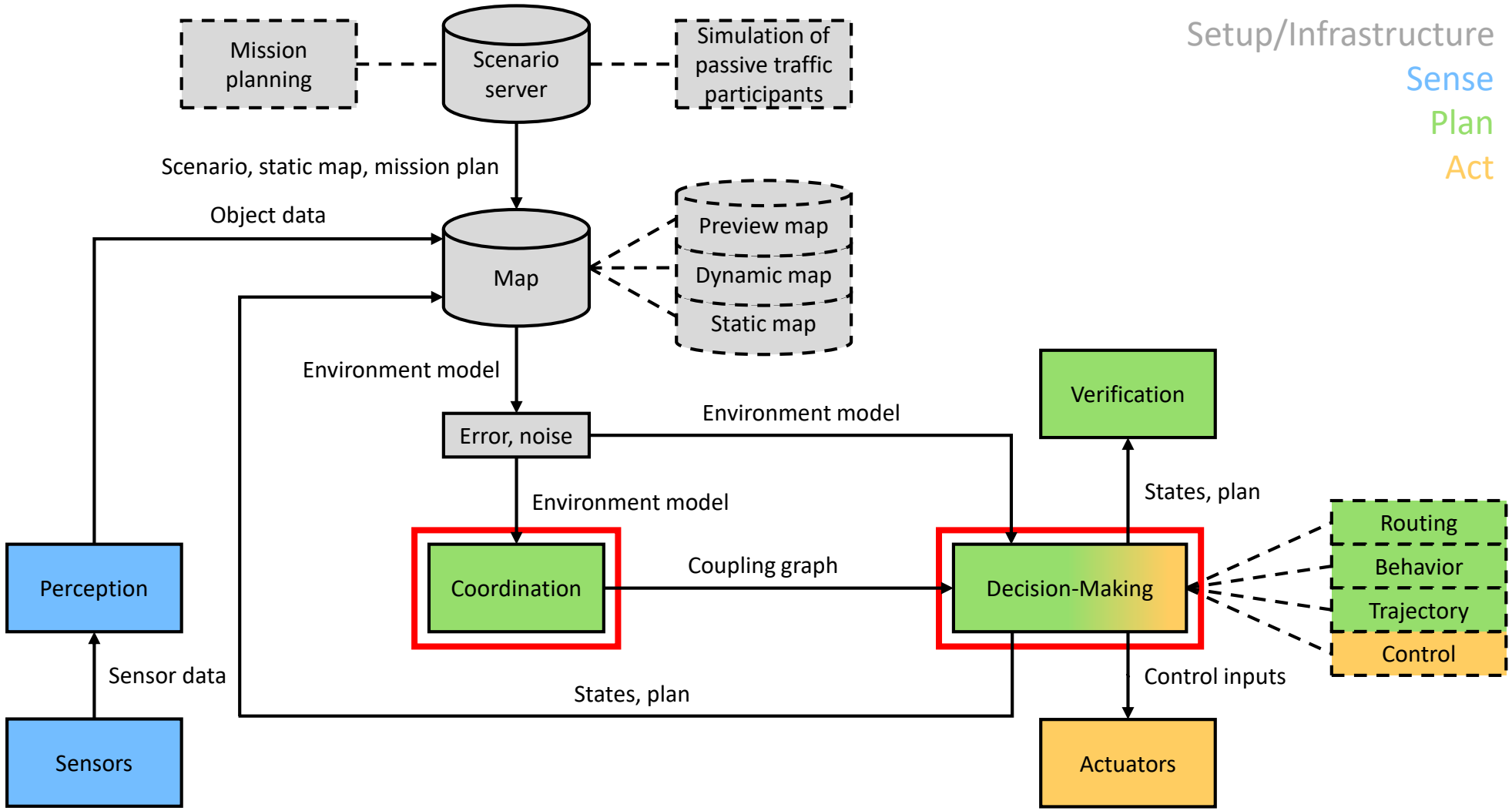
- Dynamic vehicle models
- Control and optimization
- Network and distribution
- Software architectures and testing concepts



CPM Lab architecture



CPM Lab architecture



Literature

- J. Lunze. Control Theory of Digitally Networked Dynamic Systems. Springer, 2014.

Further literature (1)

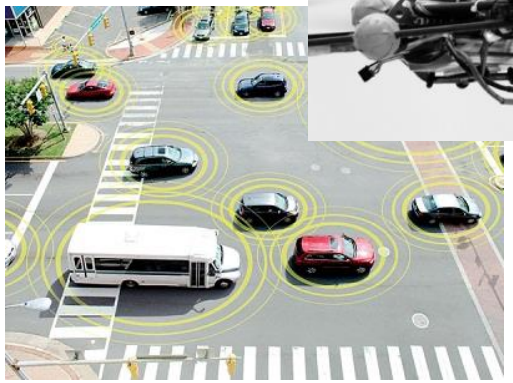
- J. Lunze. Networked Control of Multi-Agent Systems. Bookmundo Direct, 2019

Further literature (2)

- B. Alrifaee. Networked Model Predictive Control for Vehicle Collision Avoidance. PhD thesis, RWTH Aachen University, 2017.
- B. Alrifaee, F. J. Heßeler, and D. Abel. Coordinated Non-Cooperative Distributed Model Predictive Control for Decoupled Systems Using Graphs. In IFAC Workshop on Distributed Estimation and Control in Networked Systems (NecSys), 2016.
- M. Kloock, L. Kragl, J. Maczijewski, B. Alrifaee, and S. Kowalewski. Distributed Model Predictive Pose Control of Multiple Nonholonomic Vehicles. In IEEE Intelligent Vehicles Symposium (IV), 2019.
- M. Kloock, P. Scheffe, S. Marquardt, J. Maczijewski, B. Alrifaee, and S. Kowalewski. Distributed Model Predictive Intersection Control of Multiple Vehicles. In IEEE Intelligent Transportation Systems Conference (ITSC), 2019.
- M. Kloock, P. Scheffe, L. Botz, J. Maczijewski, B. Alrifaee, and S. Kowalewski. Networked Model Predictive Vehicle Race Control. In IEEE Intelligent Transportation Systems Conference (ITSC), 2019.
- B. Alrifaee. MATLAB Simulation of Networked Model Predictive Control for Vehicle Collision Avoidance, 2017. Available: https://doi.org/10.5281/zenodo.1252992

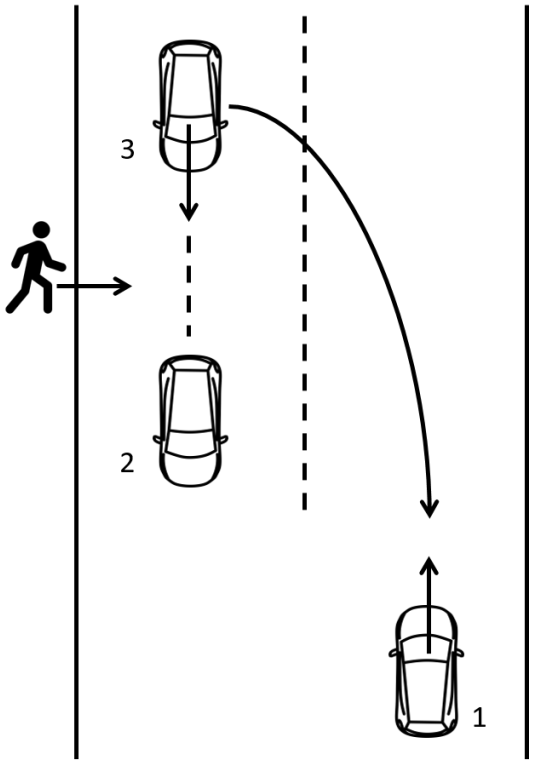
Definition of networked systems

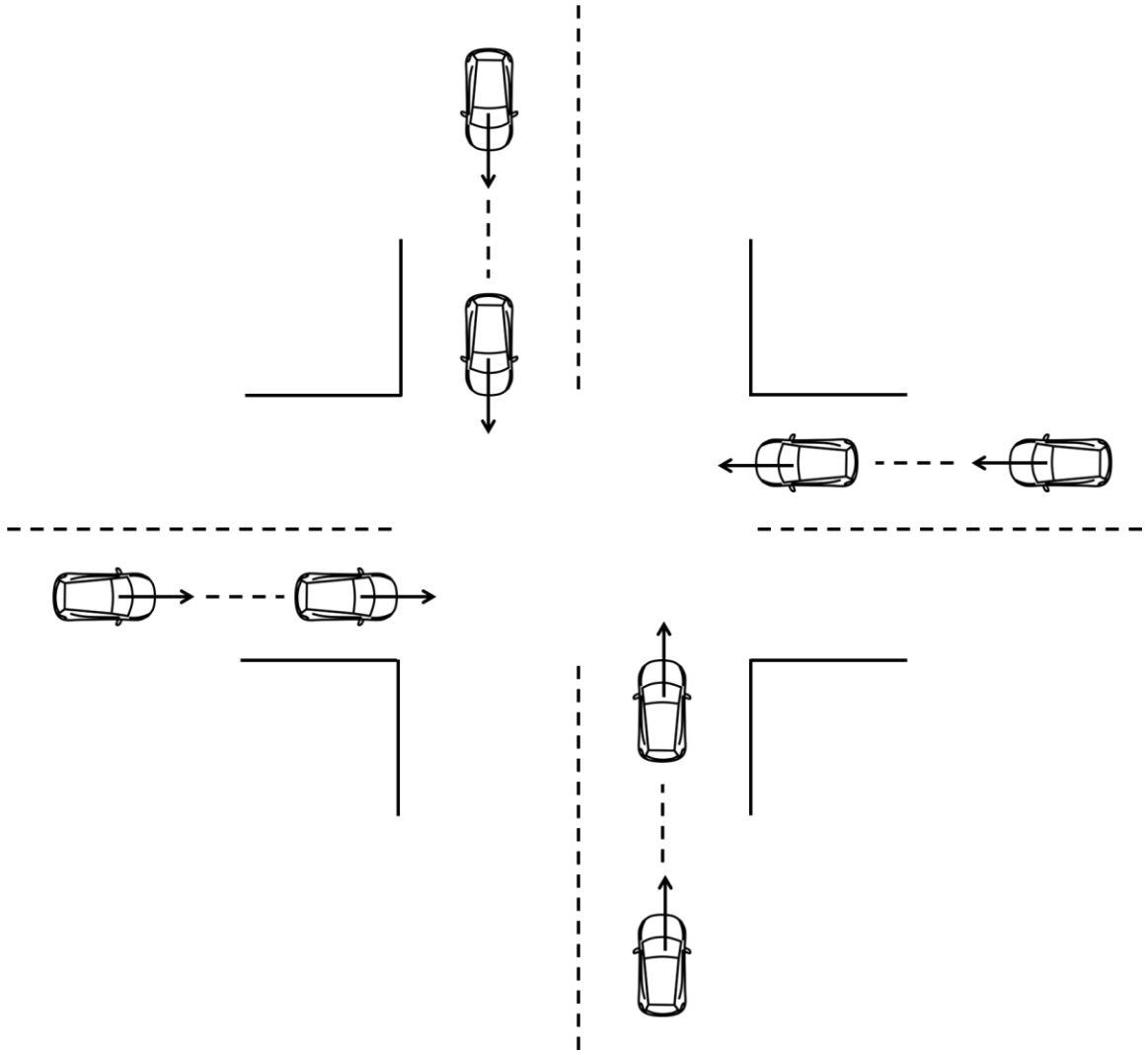
- Also called connected
- Communications
- Consist of interacting systems
- Contribution to better
  - Perception
  - Decision-making



- Many challenges arising from computation time and communications
  - Feasibility
  - Quality

Examples of contribution to better perception and decision-making



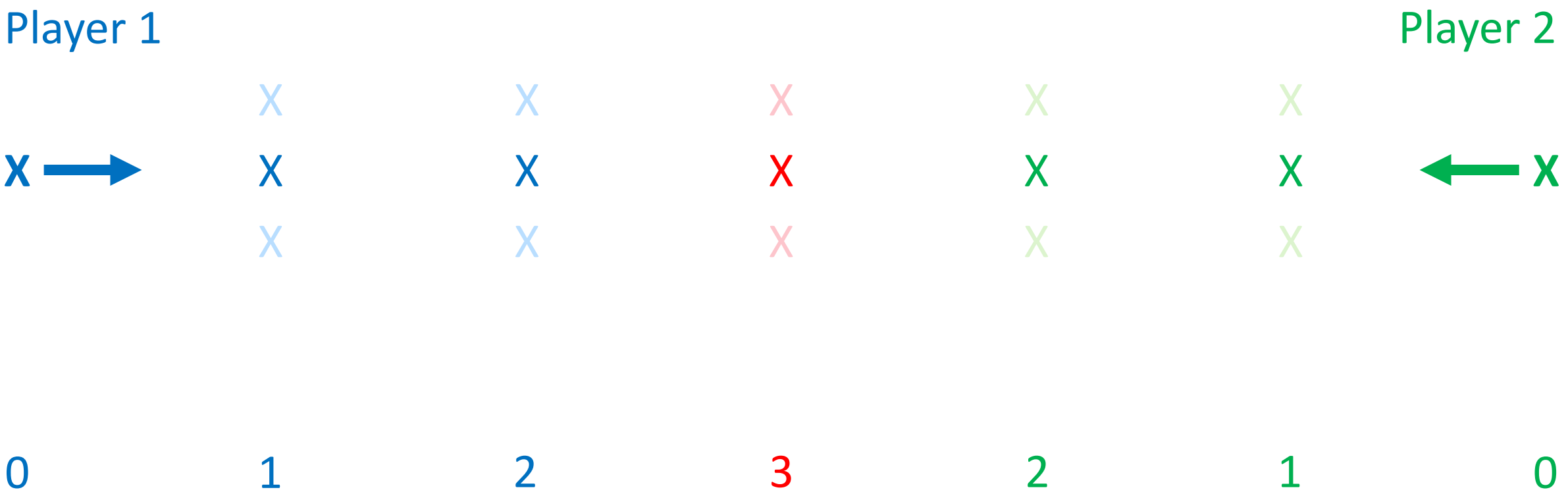


Pedestrians walking as networked MPC

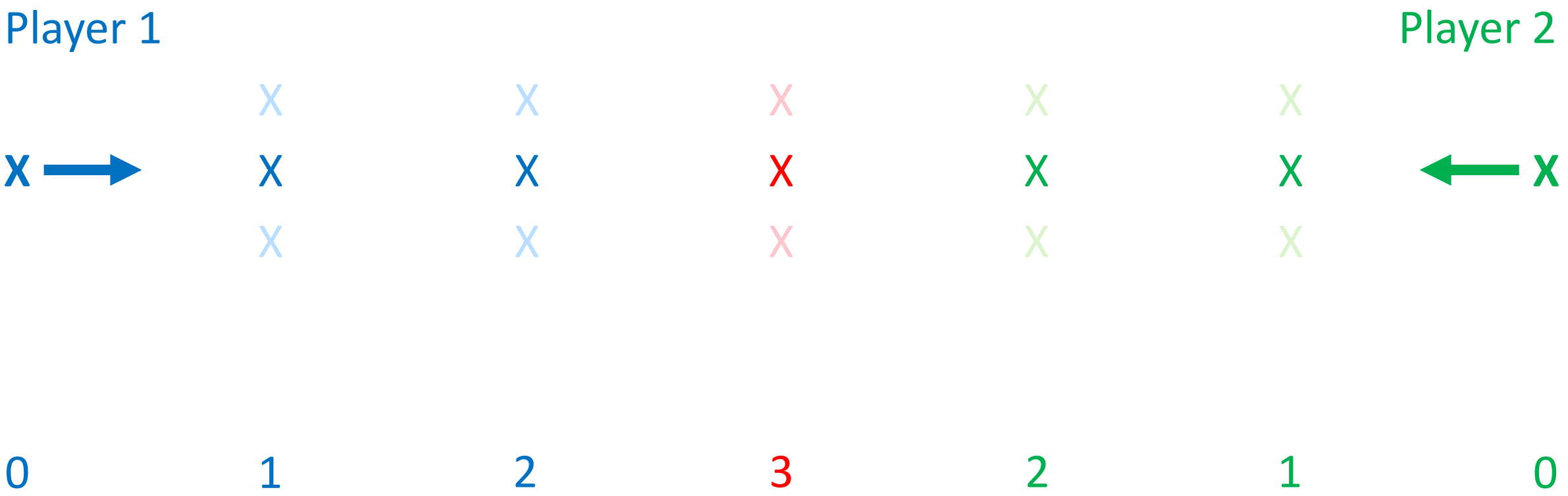
Pedestrians walking		Networked MPC	
Task	Follow the line	Reference	} Objective function
	Act as little as possible	Control input	
	Consider physical and logical limits	Constraints	
Plan	Use discrete model to anticipate your free movement for 2 steps	Predict, prediction horizon	
	Consider your physical (body) limits	Model as a constraint	
	Consider physical and logical limits of your • Action • Position • Orientation • **Do not collide with each other**	Inputs' and states' constraints	
	Plan your action for 1 step	Control horizon	

- Plan, communicate plan, act

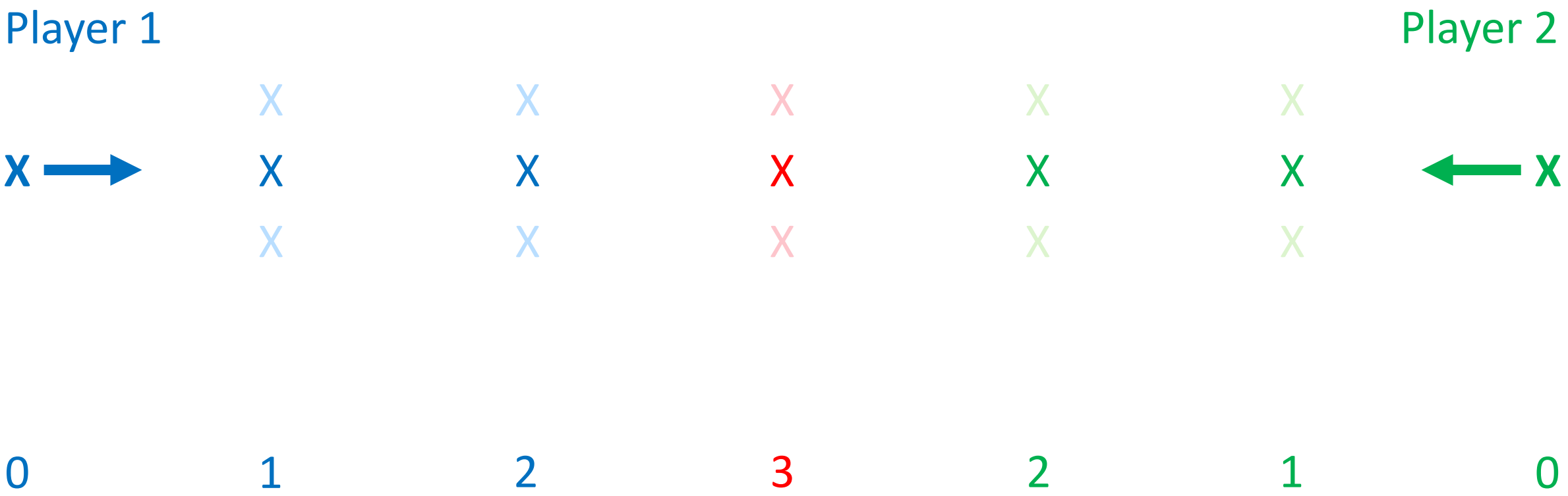
Pedestrians walking game 1



Pedestrians walking game 2



Pedestrians walking game 3



Pedestrians walking

- Two pedestrians approach each other
- Avoid each other once
- Avoid each other twice
- .
- .
- .
- (Collide together)

- n-pedestrians

Shibuya crossing

Beauty contest game

- Setup
  - Choose number between 0 and 100
  - Winner = Closest to 1/2 of average

Shamma, course on game theory and distributed control, 2019

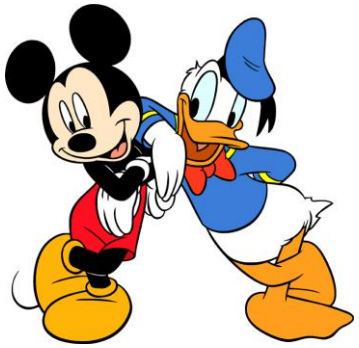
Beauty contest game

- Setup
  - Choose number between 0 and 100
  - Winner = Closest to 1/2 of average
- Decision quality of individuals affected by decisions of others

Shamma, course on game theory and distributed control, 2019

Beauty contest game

- Setup
  - Choose number between 0 and 100
  - Winner = Closest to 1/2 of average
- Decision quality of individuals affected by decisions of others
- Why “beauty contest”?


- Favorite characters = ???
- Compare: stock market

Shamma, course on game theory and distributed control, 2019

COVID-19 pandemic

$$x(t_0) = 0 \,\text{infected}$$

$$N = 80\text{M}\,\text{number of population}$$

$$i = 1, \ldots, N \,\text{population}$$

$$u^{(i)}(t) \in \{0, 1\}$$

$$x(t+1) = x(t) + R_{in}x(t) - R_{out}x(t), \text{ where } R_{in}x(t) = \sum_{i=1}^{N} u^{(i)}(t)$$

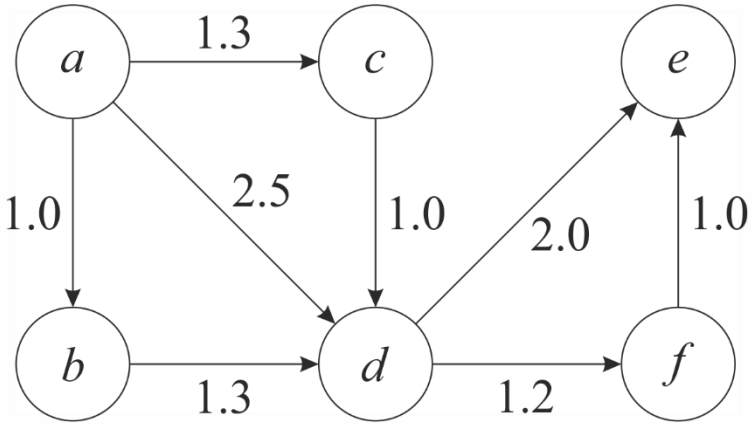
$$\text{Task: } \max_{u(t)} \sum_{i=1}^{N} u^{(i)}(t), \text{ s.t. } R_{intensive}x(t) \leq 3,000\,, \forall t$$

COVID-19 pandemic – Discussion

- COVID-19 pandemic as MPC
  - Discuss the effect of the prediction horizon

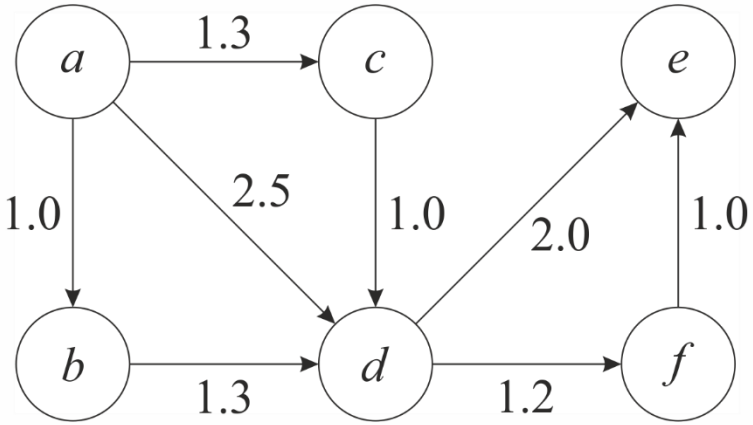
Graph theory

- Powerful tool for modeling and analyzingnetworked systems
- Reading
  - B. Alrifaee. Networked Model Predictive Control for Vehicle Collision Avoidance. PhD thesis, RWTH Aachen University, 2017.
    - Section 2.2, pages 5-7
- Definitions



Graph theory – example

- Dijkstra's algorithm for finding the shortest paths between nodes in a graph
- **Input:** graph with positive edge weights and a starting vertex a
- **Output:** shortest paths between a and all other reachable nodes from a



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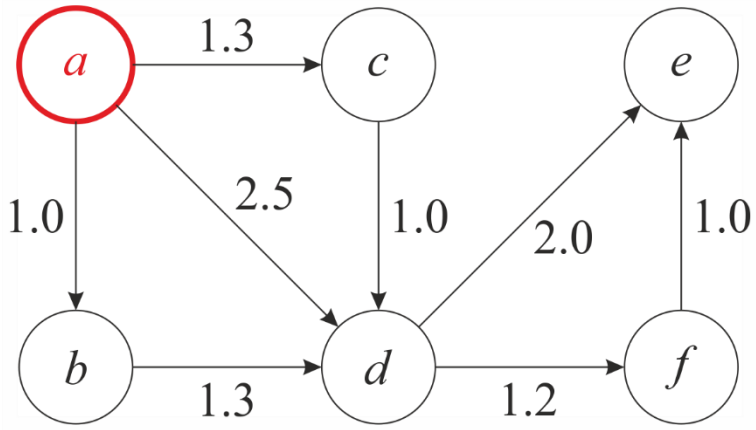
Vertex	Done	Distance	From
a	Yes	0	
b	No	∞	
c	No	∞	
d	No	∞	
e	No	∞	
f	No	∞	



$Q = []$

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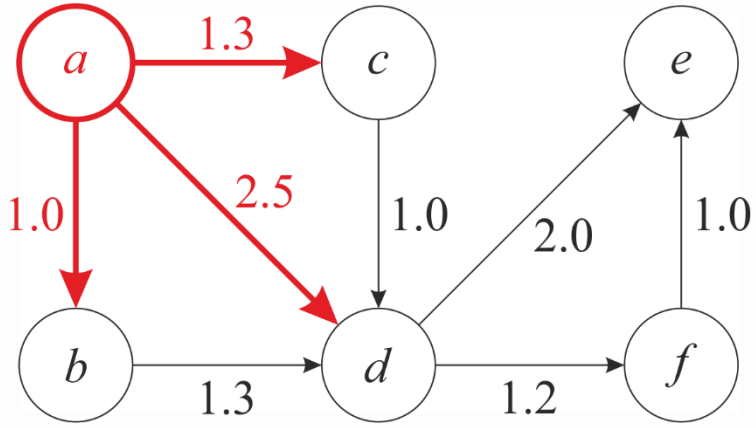
b	No	1.0	a
c	No	1.3	a
d	No	2.5	a
e	No	∞	
f	No	∞	



$Q = [b, c, d]$

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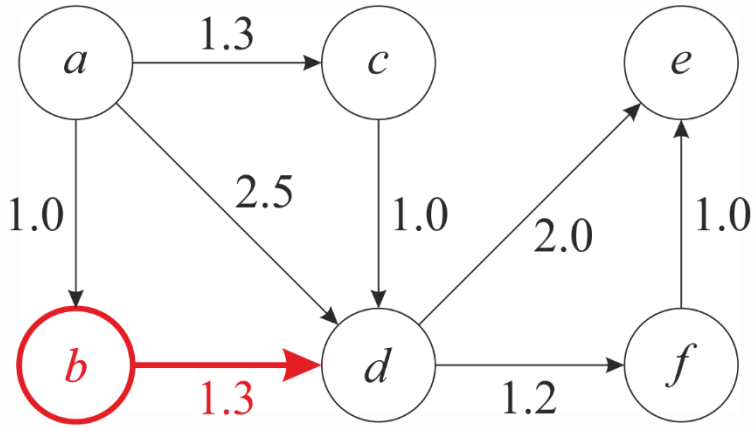

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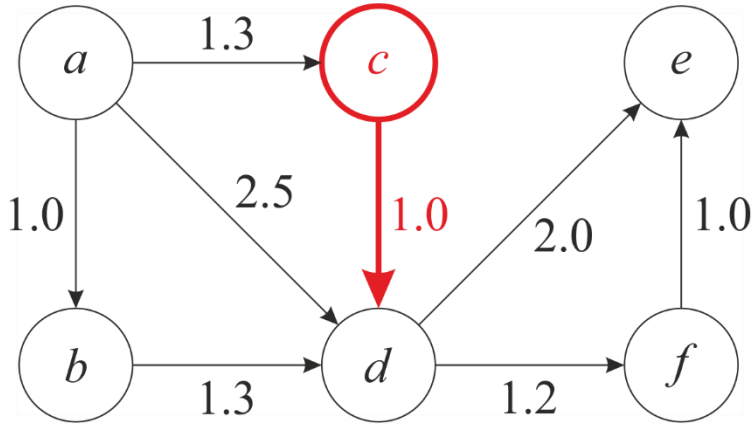

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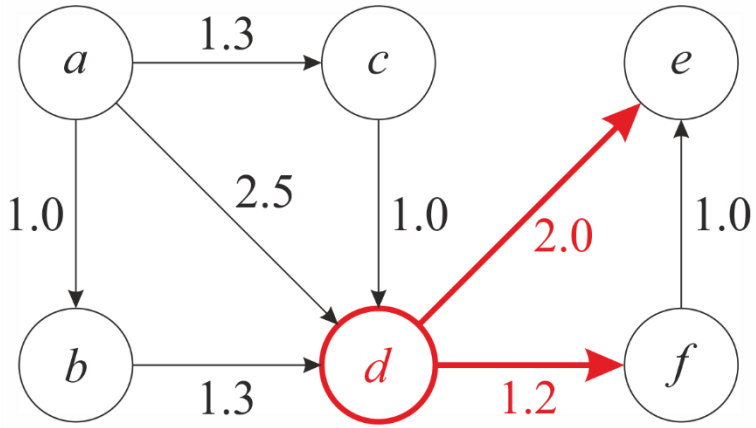
e	No	4.3	d
f	No	3.5	d



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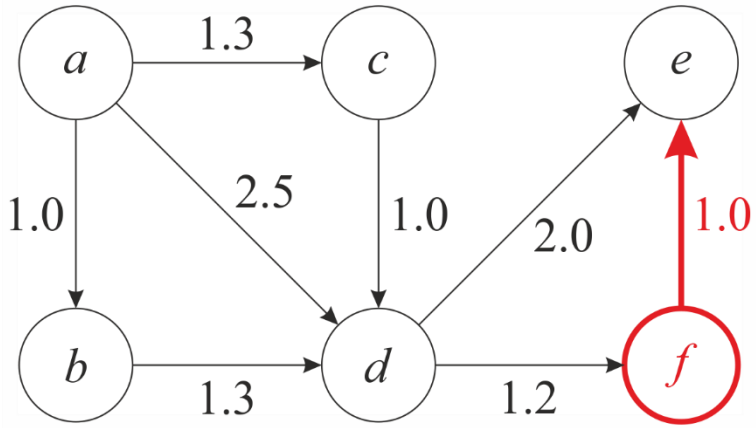

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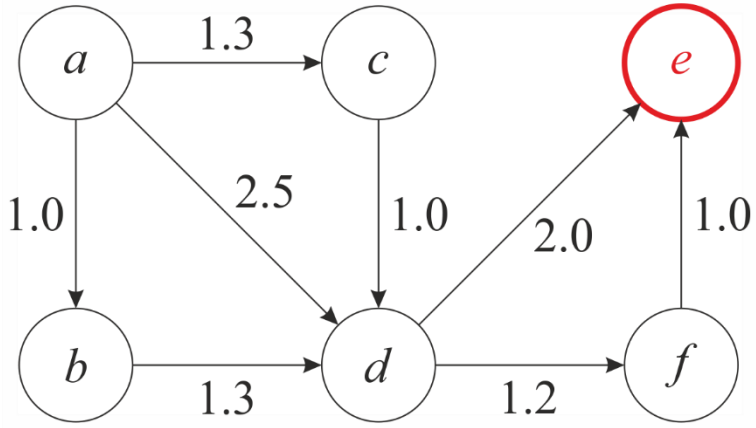

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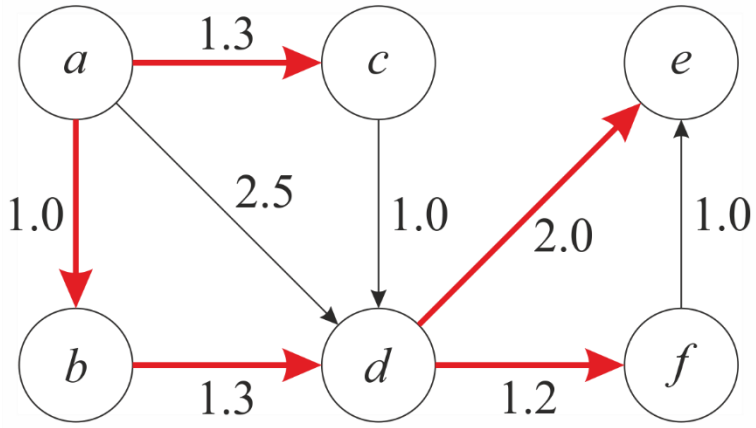


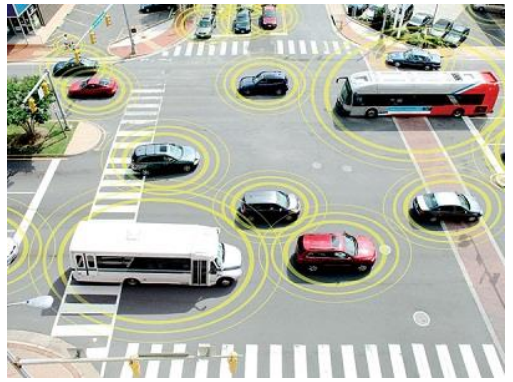
Computation: $O(N \log(N) + M)$

Networked model predictive control

- Reading
  - B. Alrifaee. Networked Model Predictive Control for Vehicle Collision Avoidance. PhD thesis, RWTH Aachen University, 2017.
    - Chapter 3, pages 20-51

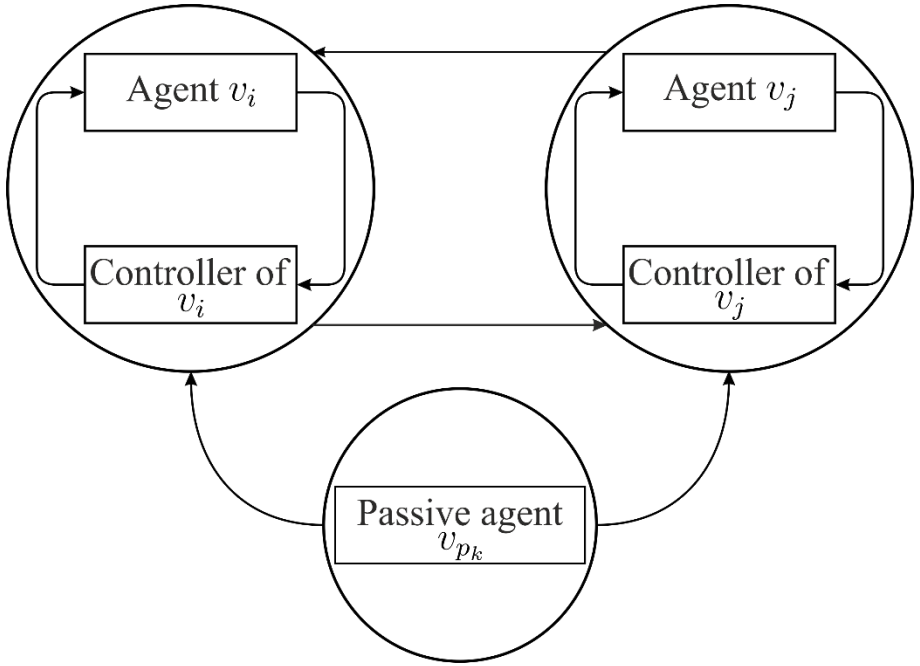
Networked control systems (NCS)

- NCS consist of interacting agents (dynamic subsystems)

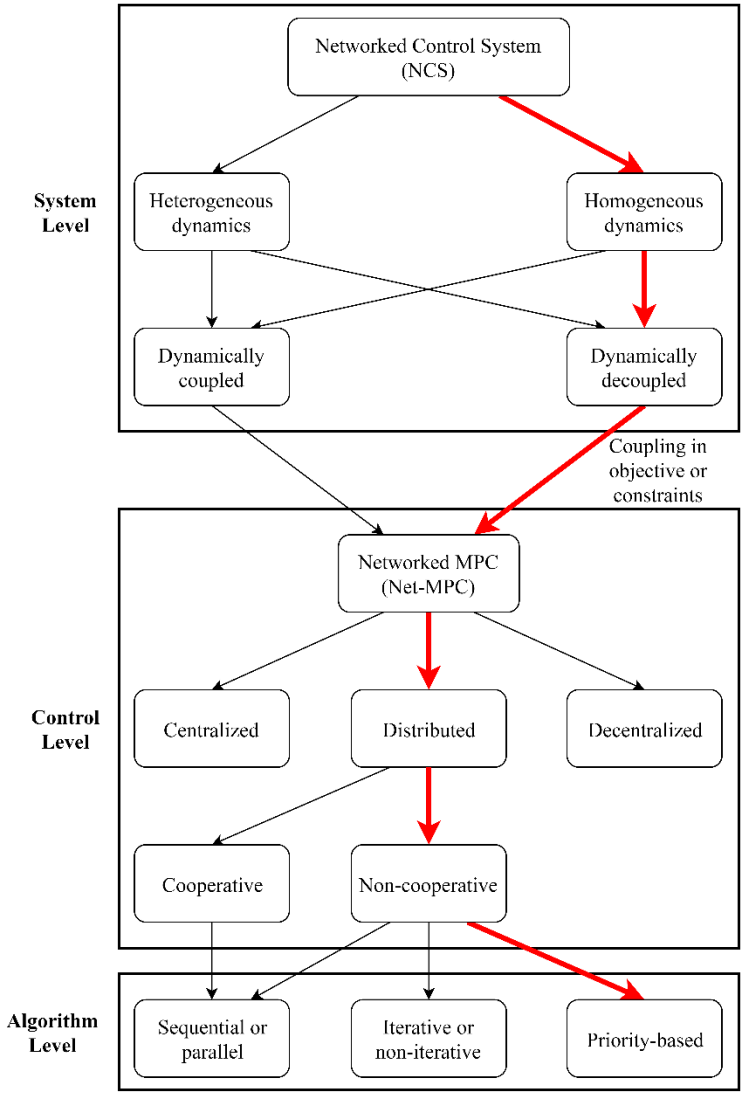


NCS

- Passive agents
  - Dynamic subsystems without networked control
  - Data communications to active agents
- (Active) Agents
  - Data exchange
  - Achieve their goals while taking the interaction with other agents into consideration
  - Full knowledge about passive agents and their future states
- Classification of agents as a step to full automation of NCS and consideration of non-automatable agents
- Communications restrictions, e.g., time delays, and computation time affect stability and performance
- Network: time-invariant or time-variant



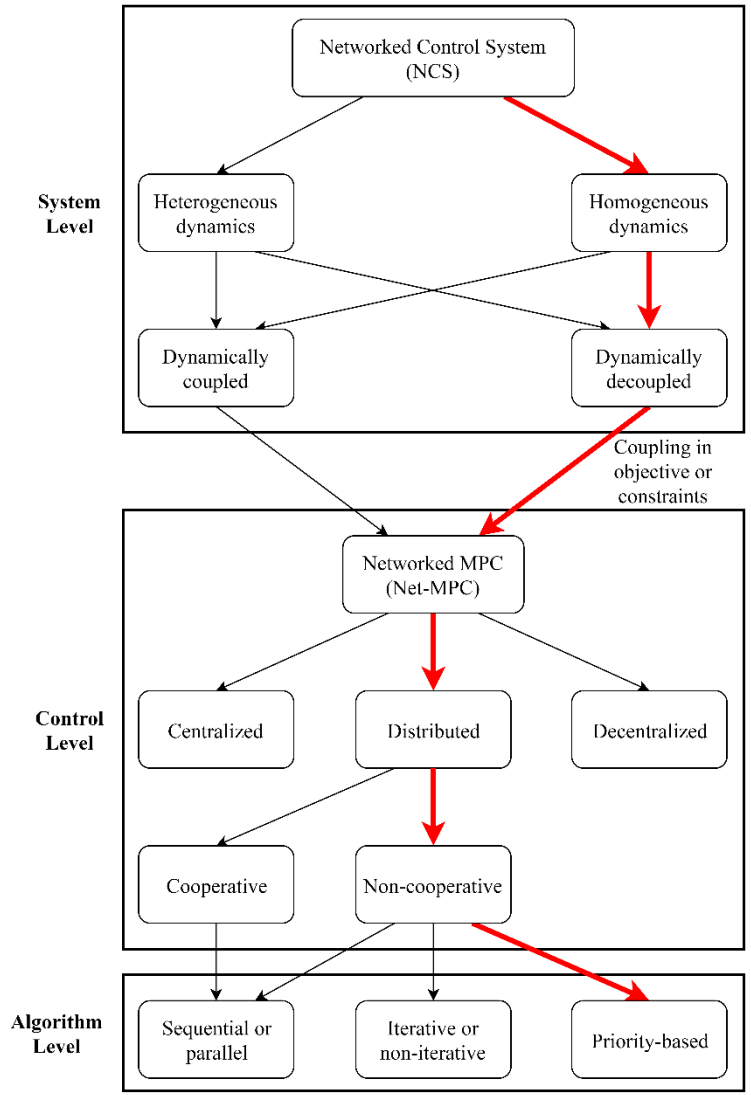
NCS classification



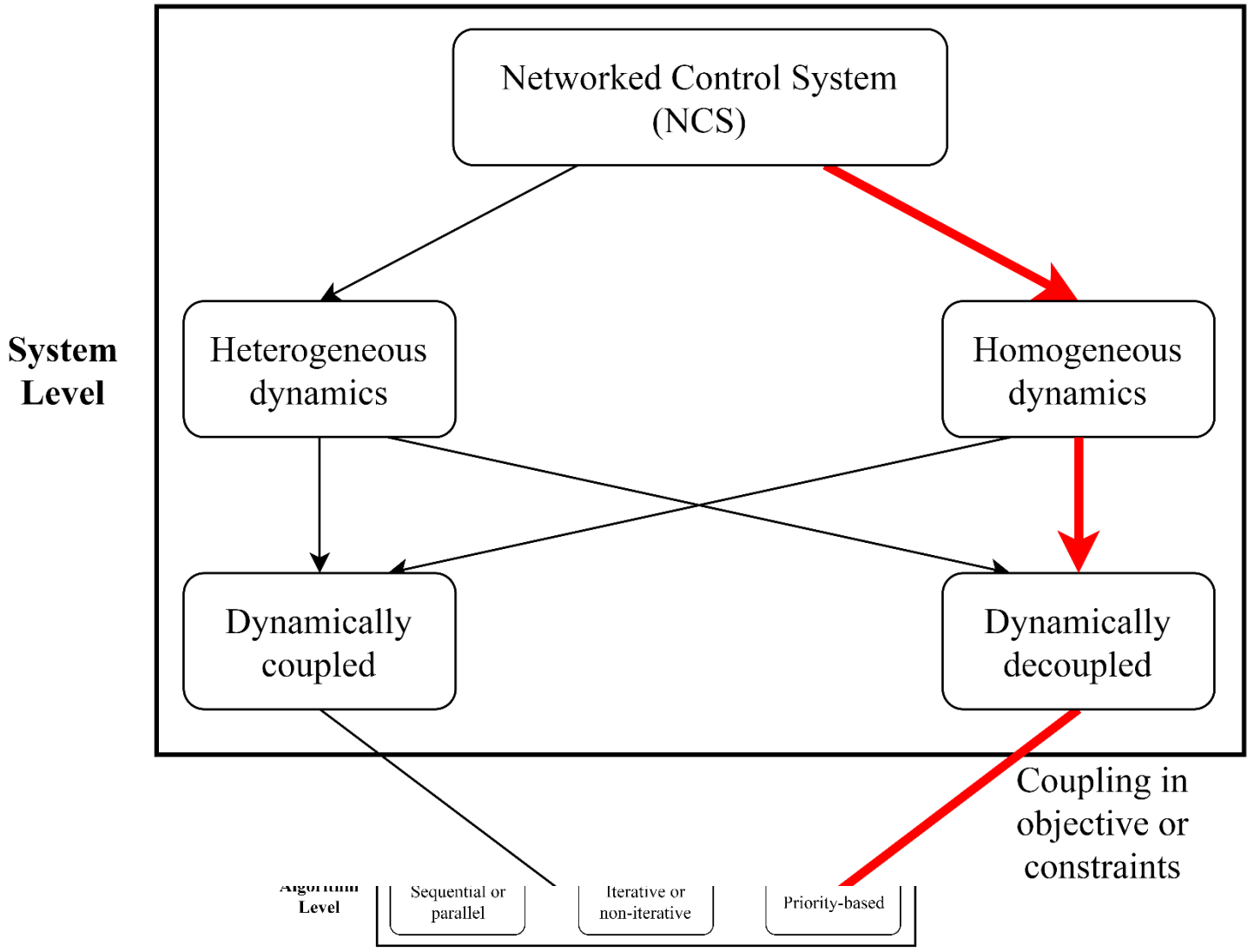
NCS classification

- *Control strategy*: combination of a control method and the algorithm applied to it
- Selection of control strategy based on:
  - NCS categories in the system level
  - Available computation time
  - Communications requirements
- *Computation time*: time required for the whole NCS to reach a solution at a given time step, i.e.,
  - Measure the states
  - Formulate and solve the optimization problem
  - Apply the inputs to all agents
  - Communications of required data

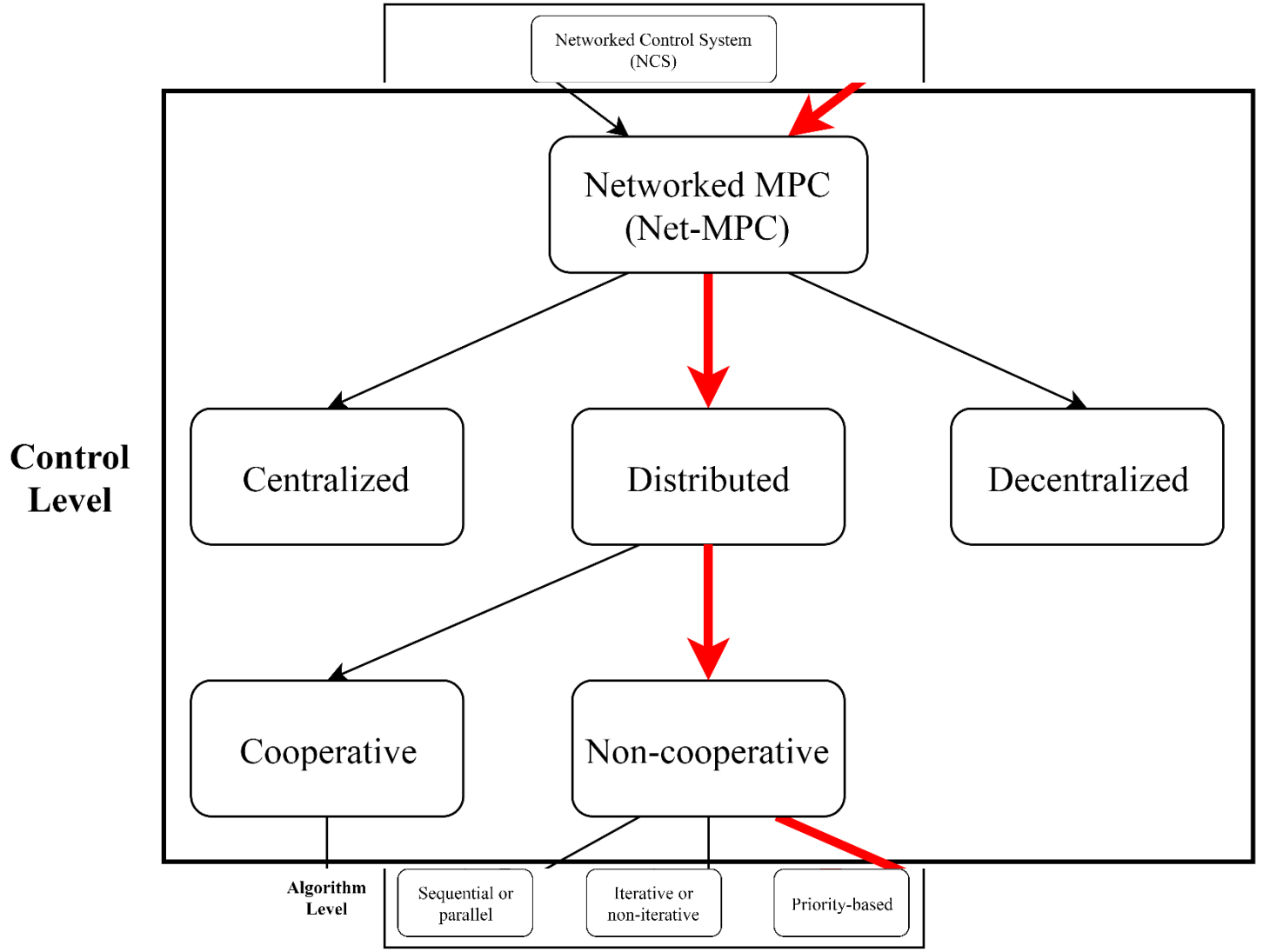
  Sequence depends on the control strategy



NCS classification



NCS classification



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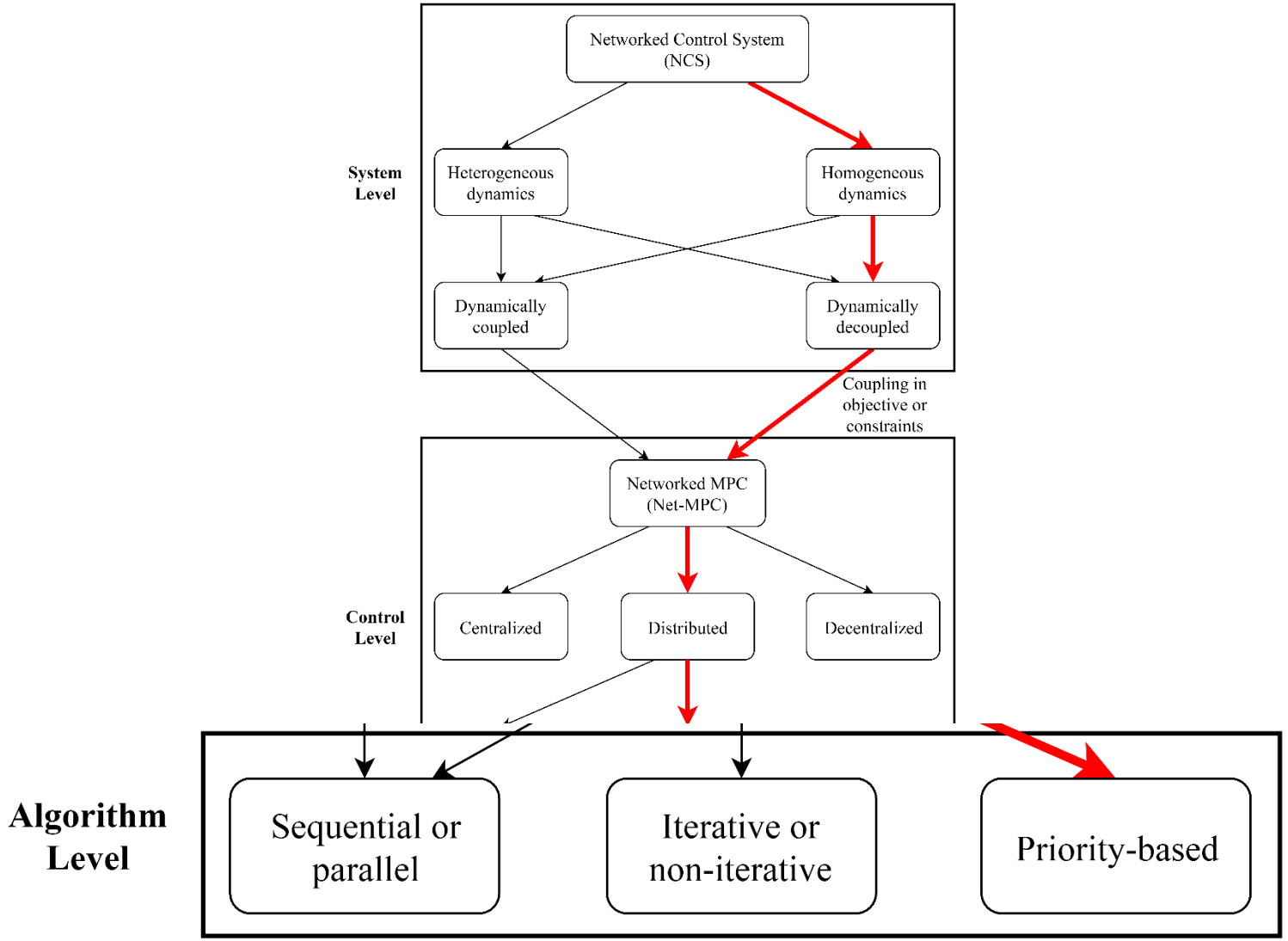


Networked model predictive control

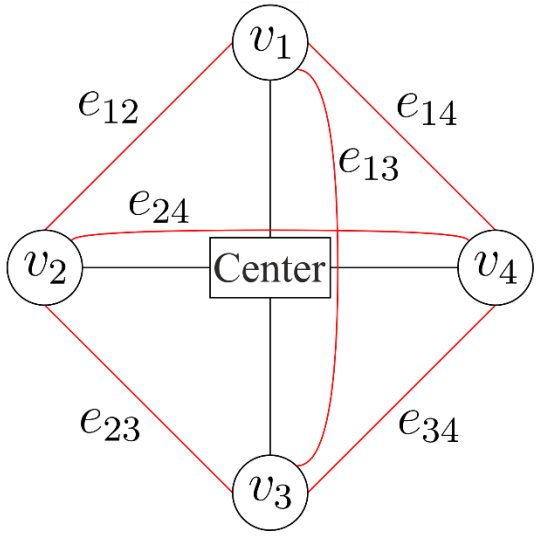
- Method
  - Reducing computation time and communications
  - Enhancing feasibility and quality of control

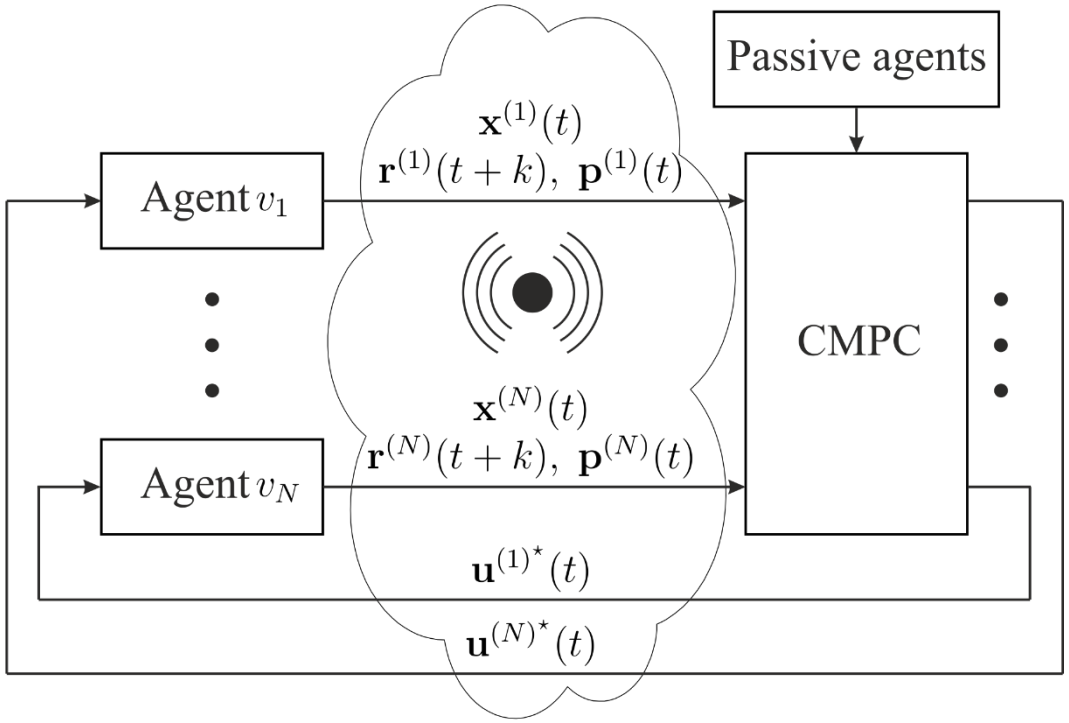
Formulation of Net-MPC: basic formulation

- Advantage of Net-MPC
  - Dealing with the action of other agents with respect to their future intention, while making decisions of the agent's own control actions

$$
\begin{aligned}
J^{(i)\star} = & \min_{\Delta \mathbf{u}^{(i)}(\cdot)} \\
& \sum_{k=1}^{H_p-1} l_x^{(i)}(\mathbf{x}^{(i)}(t+k), \mathbf{r}^{(i)}(t+k)) + l_{x_{H_p}}^{(i)}(\mathbf{x}^{(i)}(t+H_p), \mathbf{r}^{(i)}(t+H_p)) + \\
& \sum_{k=0}^{H_u-1} l_u^{(i)}(\Delta \mathbf{u}^{(i)}(t+k)) + \sum_{\substack{j \\ v_j \in \mathcal{V}^{(i)}}} \sum_{k=1}^{H_p} c_o^{(i,j)}(\mathbf{x}^{(i)}(t+k), \mathbf{x}^{(j)}(t+k)) \\
& \text{subject to } (\forall v_j \in \mathcal{V}^{(i)},\ \forall v_p \in \mathcal{V}_p^{(i)}) : \\
& \mathbf{x}^{(i)}(t+1+k) = f^{(i)}(\mathbf{x}^{(i)}(t+k), \mathbf{u}^{(i)}(t+k)),\ k = 0, \ldots, H_p - 1 \\
& \mathbf{x}^{(i)}(t+k) \in \mathcal{X}^{(i)},\ k = 1, \ldots, H_p - 1 \\
& \mathbf{x}^{(i)}(t+H_p) \in \mathcal{X}_{H_p}^{(i)} \\
& \mathbf{u}^{(i)}(t+k) \in \mathcal{U}^{(i)},\ k = 0, \ldots, H_u - 1 \\
& \Delta \mathbf{u}^{(i)}(t+k) \in \Delta \mathcal{U}^{(i)},\ k = 0, \ldots, H_u - 1 \\
& c_c^{(i,j)}(\mathbf{x}^{(i)}(t+k), \mathbf{x}^{(j)}(t+k)) \leq 0,\ k = 1, \ldots, H_p \\
& c_c^{(i,p)}(\mathbf{x}^{(i)}(t+k), \mathbf{x}^{(p)}(t+k)) \leq 0,\ k = 1, \ldots, H_p
\end{aligned}
$$

Net-MPC: centralized MPC



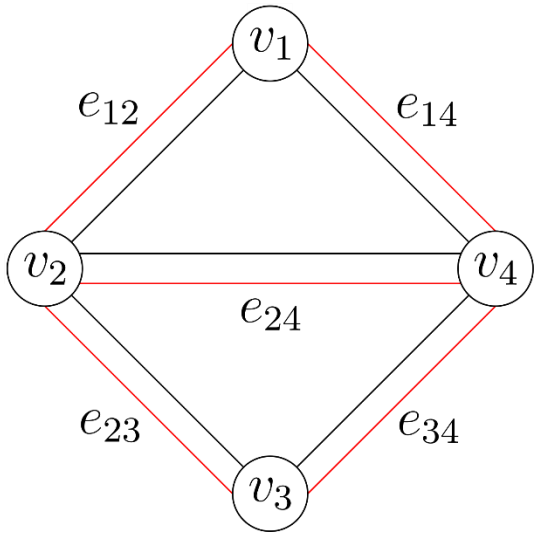


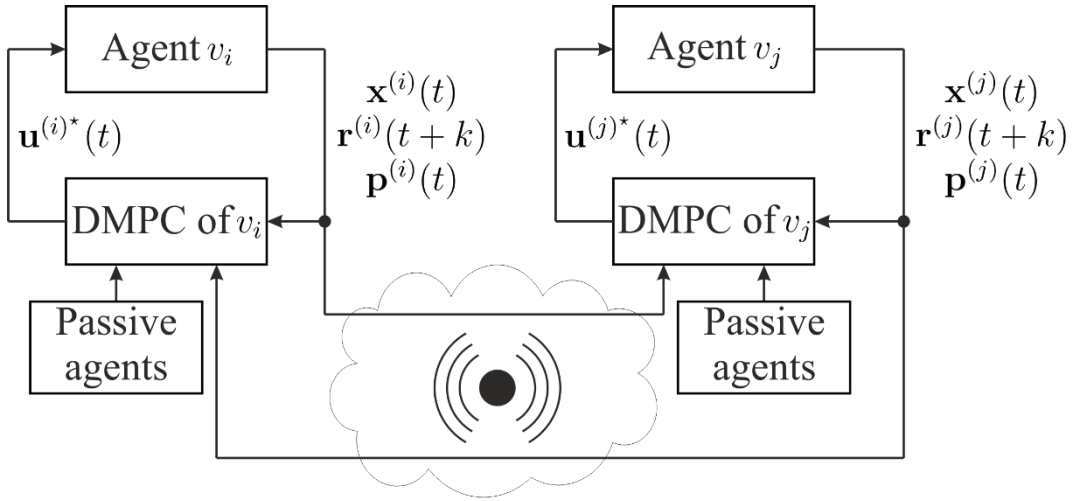
- Not applicable in practice due to
  - High computation time
  - Safety hazards
- Benchmark for comparing different distributed MPC strategies

Net-MPC: terms

- NCS performance
  - NCS stability, solution feasibility, solution optimality, solution quality
- A NCS is ***stable*** if each of its agents is stable in the network
- A solution is ***agent-feasible*** if each agent's controller generates a feasible solution in terms of its own optimization problem
- A solution is ***NCS-feasible*** if it is feasible in terms of a corresponding CMPC
- A solution is ***agent-optimal*** if each single agent's controller generates an optimal solution in terms of its own optimization problem
- A solution is ***NCS-optimal*** if it is optimal in terms of a corresponding CMPC
- The ***NCS-quality*** is defined as the quality of a solution compared with the solution of CMPC
- Assumption: a solution to CMPC exists and it is NCS-stable, -feasible, and -optimal

Net-MPC: cooperative distributed MPC





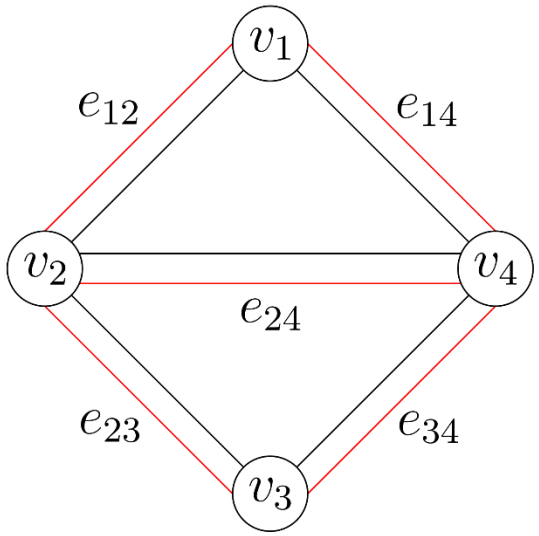
- Decomposition of centralized MPC into smaller optimization problems
- Each agent just considers hypothetical plans of its neighbors

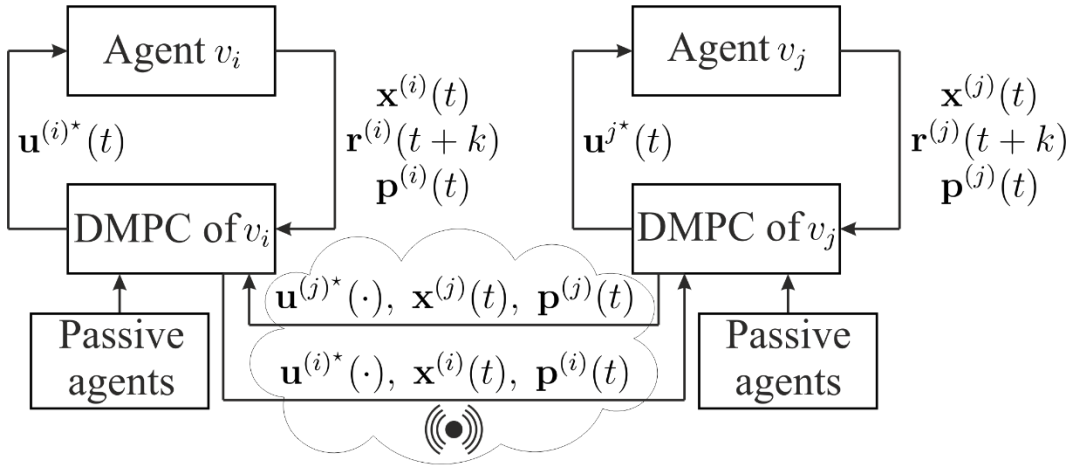
Net-MPC: cooperative distributed MPC

Prediction consistency

- Prediction consistency means that the predictions $x^{(j)}(t+k)$, $k=1,\ldots,H_p$ used or computed in the optimization problem of an agent v_i at a time instance t for an agent v_j coincide with the predictions computed by agent v_j itself at the same time instance t
- Without satisfaction of this property, no guarantee for NCS-stability and -feasibility
- Coop. DMPC does not satisfy the prediction consistency property
  - Exception: if the NCS is fully connected
  - Coop. DMPC becomes CMPC except the communications structures
  - High computation time

Net-MPC: non-cooperative distributed MPC





- Decomposition of centralized MPC into smaller optimization problems
- Consideration only of the own objective function, own constraints, and the coupling objectives and constraints with neighbors (greedy algorithm)
- Use of communicated optimized predictions from neighbors → time delay

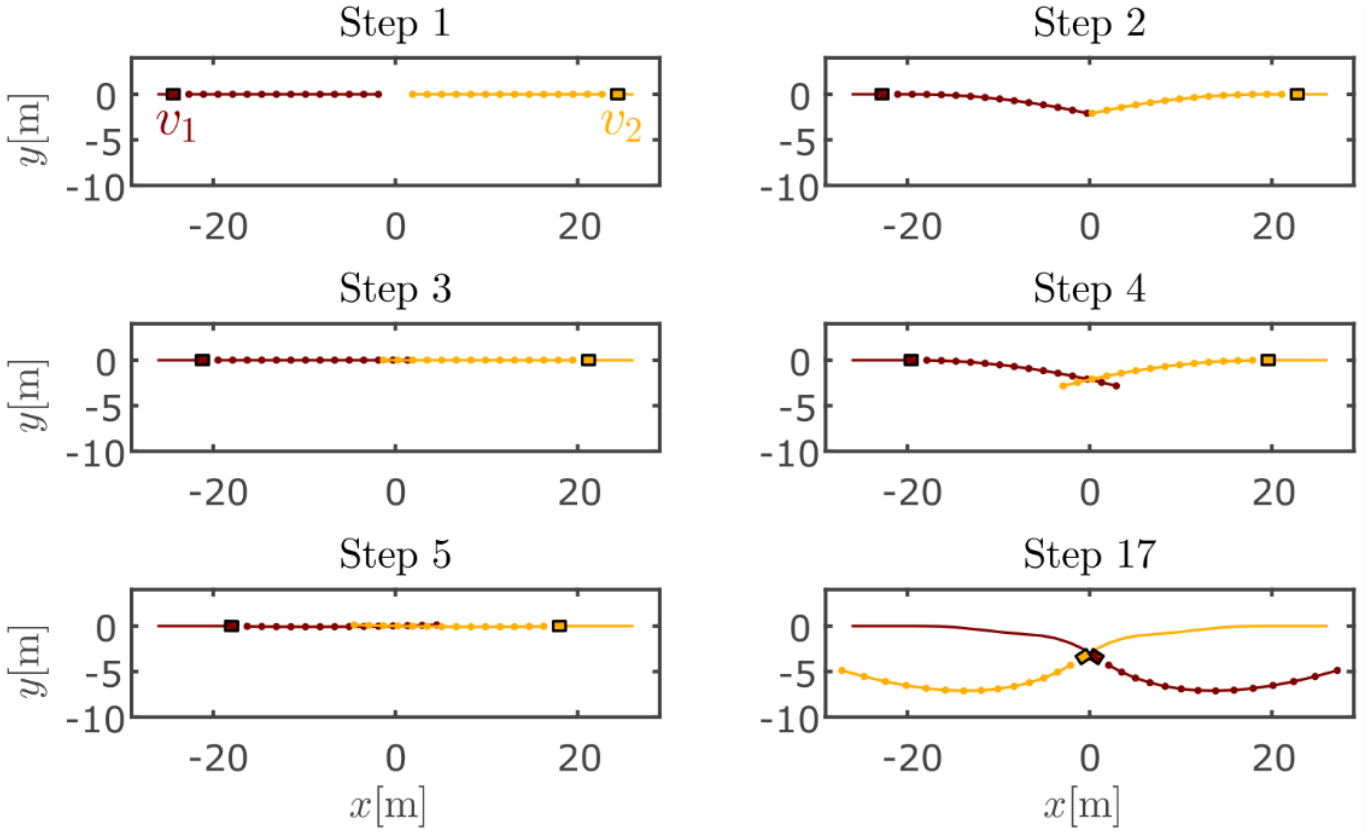
Net-MPC: non-cooperative distributed MPC

Prediction consistency

- Prediction consistency means that the predictions $x^{(j)}(t+k)$, $k=1,\ldots,H_p$ used or computed in the optimization problem of an agent v_i at a time instance t for an agent v_j coincide with the predictions computed by agent v_j itself at the same time instance t
- Without satisfaction of this property, no guarantee for NCS-stability and -feasibility
- Non-Coop. DMPC does not satisfy the prediction consistency property due to the time delay in the communications

Net-MPC: non-cooperative distributed MPC

- Vehicles move with the same velocity
- Vehicles are located symmetrically with respect to the vertical axes
- Time delay of one time step



Pedestrians walking as networked MPC

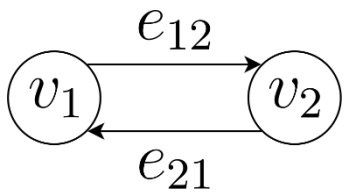
Pedestrians walking		Networked MPC	
Task	Follow the line	Reference	} Objective function
	Act as little as possible	Control input	
	Consider physical and logical limits	Constraints	
Plan	Use discrete model to anticipate your free movement for 2 steps	Predict, prediction horizon	
	Consider your physical (body) limits	Model as a constraint	
	Consider physical and logical limits of your • Action • Position • Orientation • **Do not collide with each other**	Inputs' and states' constraints	
	Plan your action for 1 step	Control horizon	

- Plan, communicate plan, act

Net-MPC: non-cooperative distributed MPC

Coupling graph

- Coupling graph contains cycles
  - Consideration of exactly the same coupling
- Cycles in coupling graph lead to loss of prediction consistency property

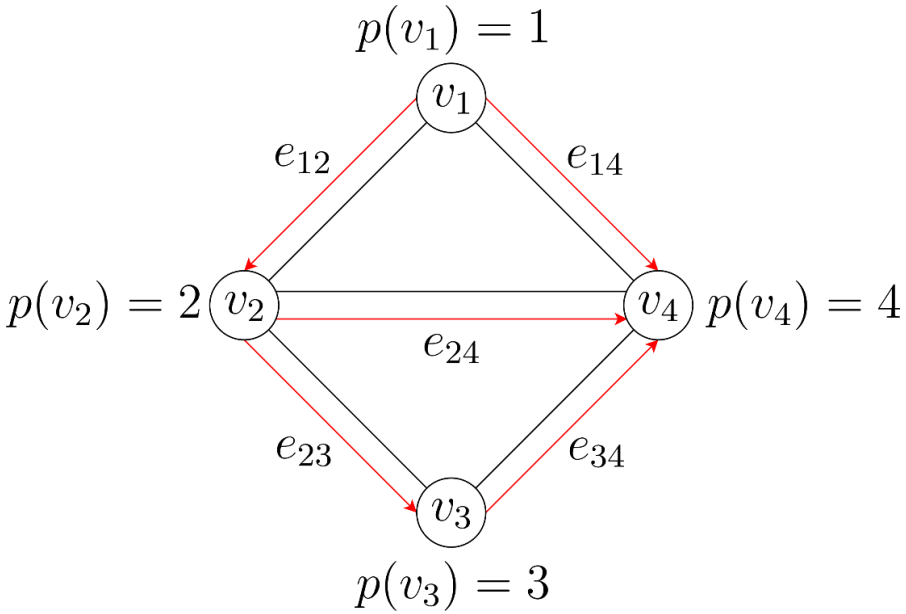


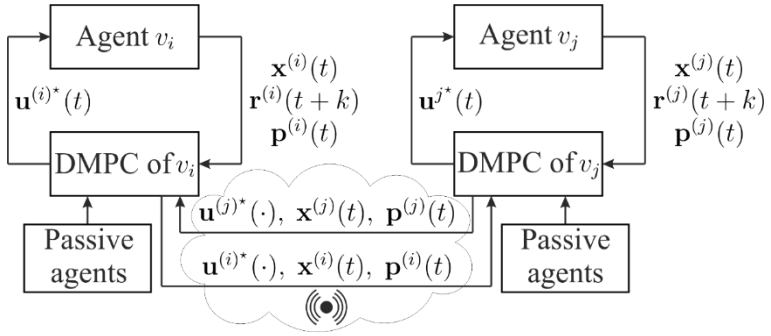
- Solutions:
  - Solve in sequence and iterate → high computation time
  - **Priority-Based Non-Cooperative Distributed MPC**

Networked model predictive control

- Reading
  - B. Alrifaee. Networked Model Predictive Control for Vehicle Collision Avoidance. PhD thesis, RWTH Aachen University, 2017.
    - Chapter 3, pages 51-77

Net-MPC: priority-based non-cooperative distributed MPC

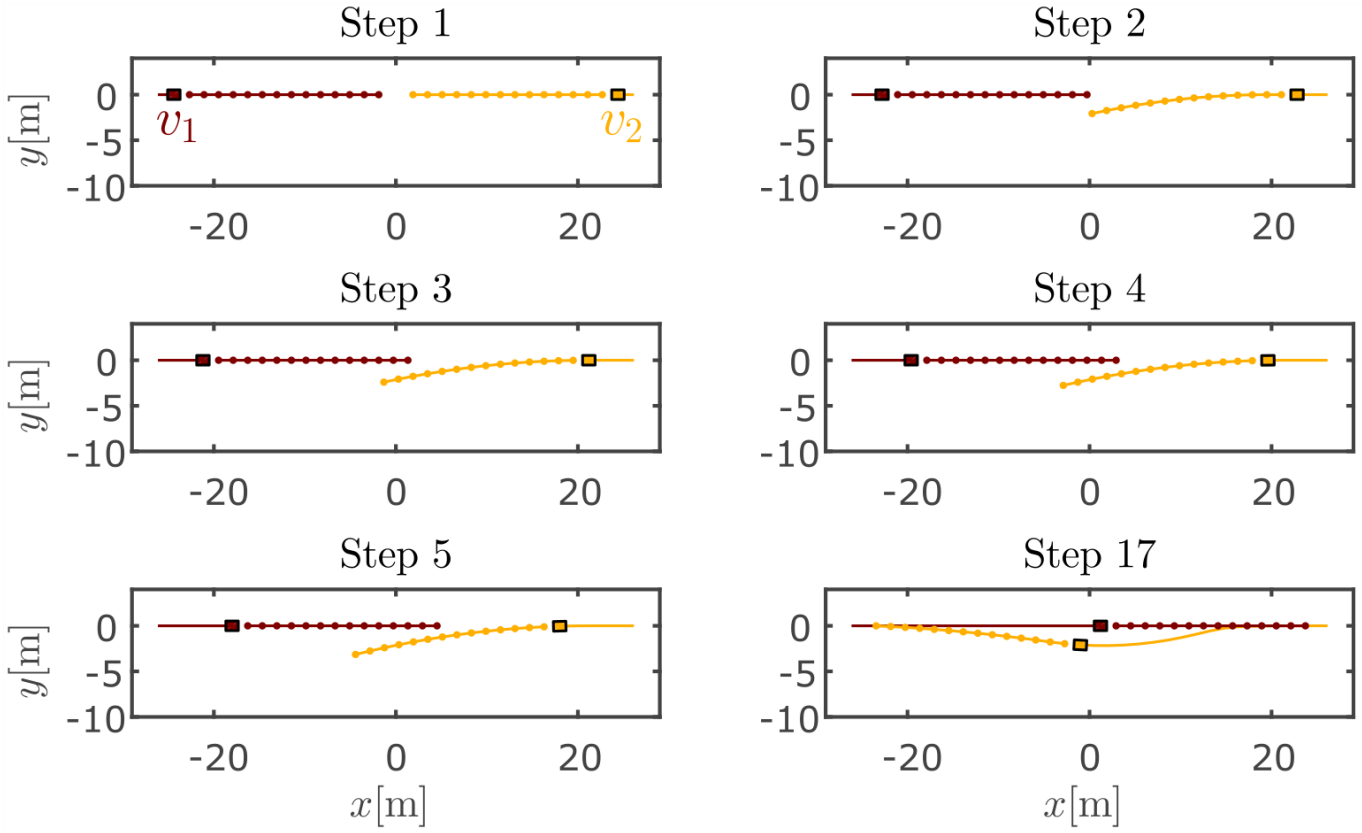




- Assign agents distinct priorities
- Lower priority value corresponds to a higher priority
- Passive agents have the highest priority
- Consideration of the own objective function, constraints, and only the coupling objectives and constraints with higher priority agents

Net-MPC: priority-based non-cooperative distributed MPC

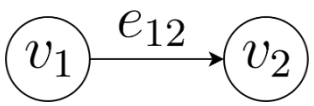
- Example: Non-Coop. DMPC vs. PB-Non-Coop. DMPC



Net-MPC: priority-based non-cooperative distributed MPC

Coupling graph

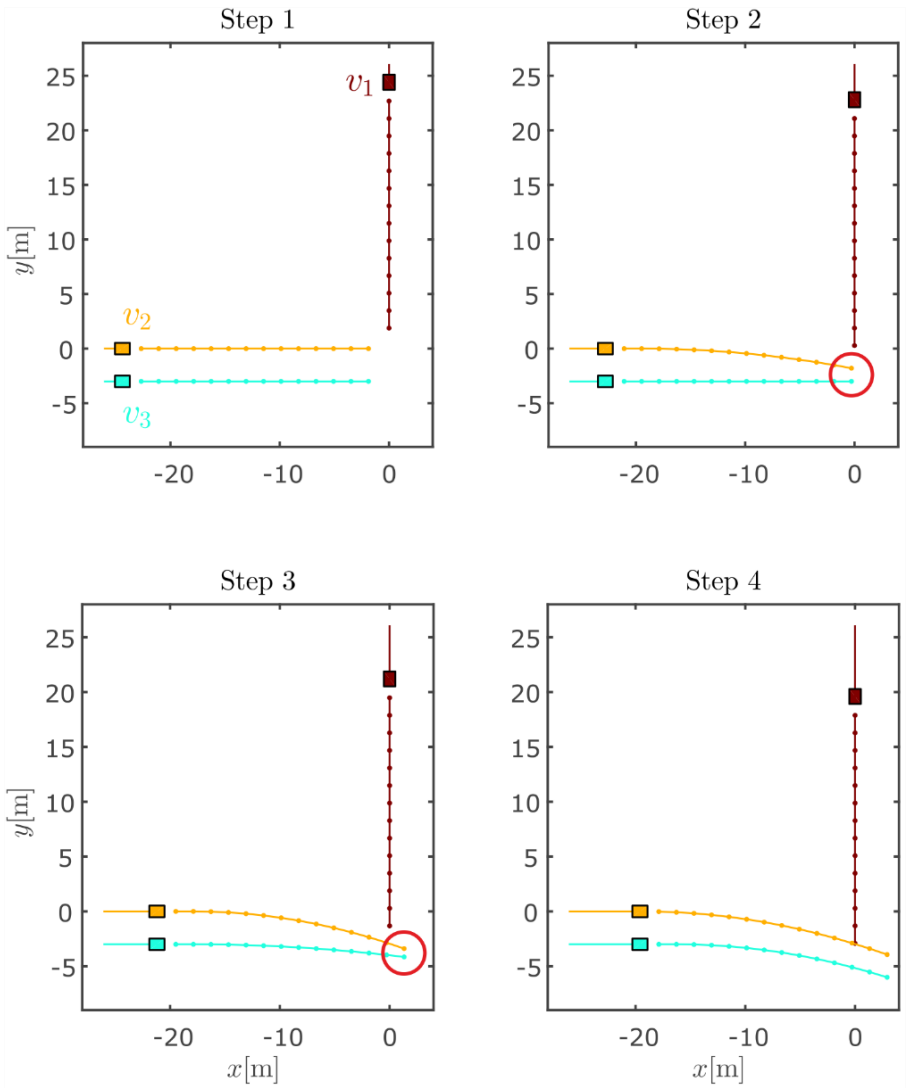
- Directed acyclic coupling graph (DAG) → proof using adjacency matrix



- Time delay of predictions of higher priority neighbors
  - Done in the case of time-invariant coupling topology and assuming bounded disturbances in higher priority neighbors
  - Loss of the prediction consistency property in the case of a time-variant coupling topology

Net-MPC: priority-based non-cooperative distributed MPC

- Vehicles move with the same velocity
- Priorities
  - v_1 First
  - v_2 Second
  - v_3 Third
- Time delay of one time step
- Solutions:
  - Infinite prediction horizon → not implementable in real-time
  - Incorporating a sequential algorithm into the PB-Non-Coop. DMPC strategy



Net-MPC: priority-based non-cooperative distributed MPC

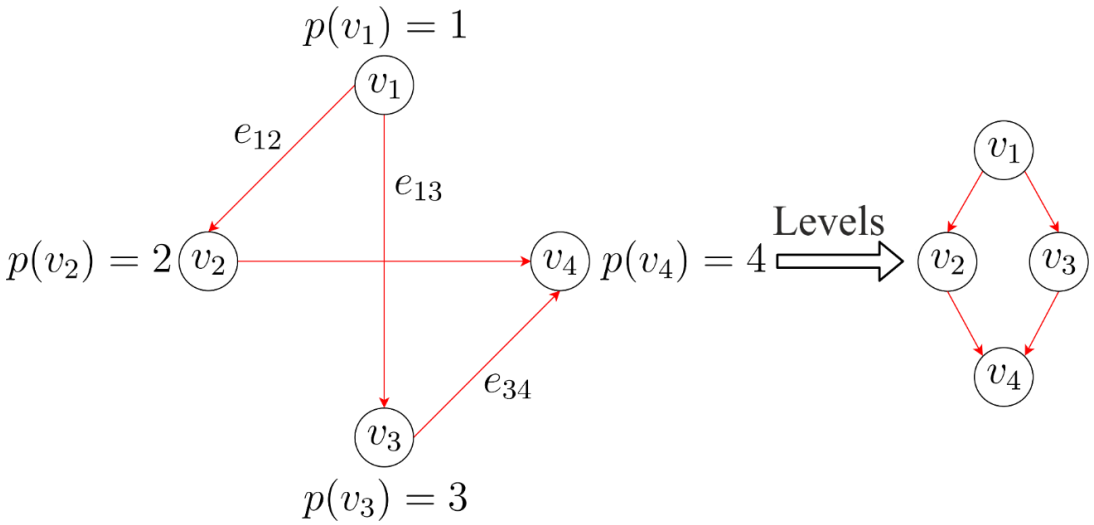
Sequential algorithm based on coupling graph

- Priorities generate a partial order
- Converting the partial order into a topological order
  - Renumbering the vertices of the coupling graph with their corresponding priorities
  - Valid sequence for solving the optimization problems
  - Possible if the coupling graph is DAG (proven)
- Our topological order is not unique → parallelization of subsets of the optimization problems possible

Net-MPC: priority-based non-cooperative distributed MPC

Algorithm to determine parallelizable vertices

- **Input:** adjacency matrix of a DAG
- **Output:** parallelizable vertices saved in a matrix $\mathbf{L}$

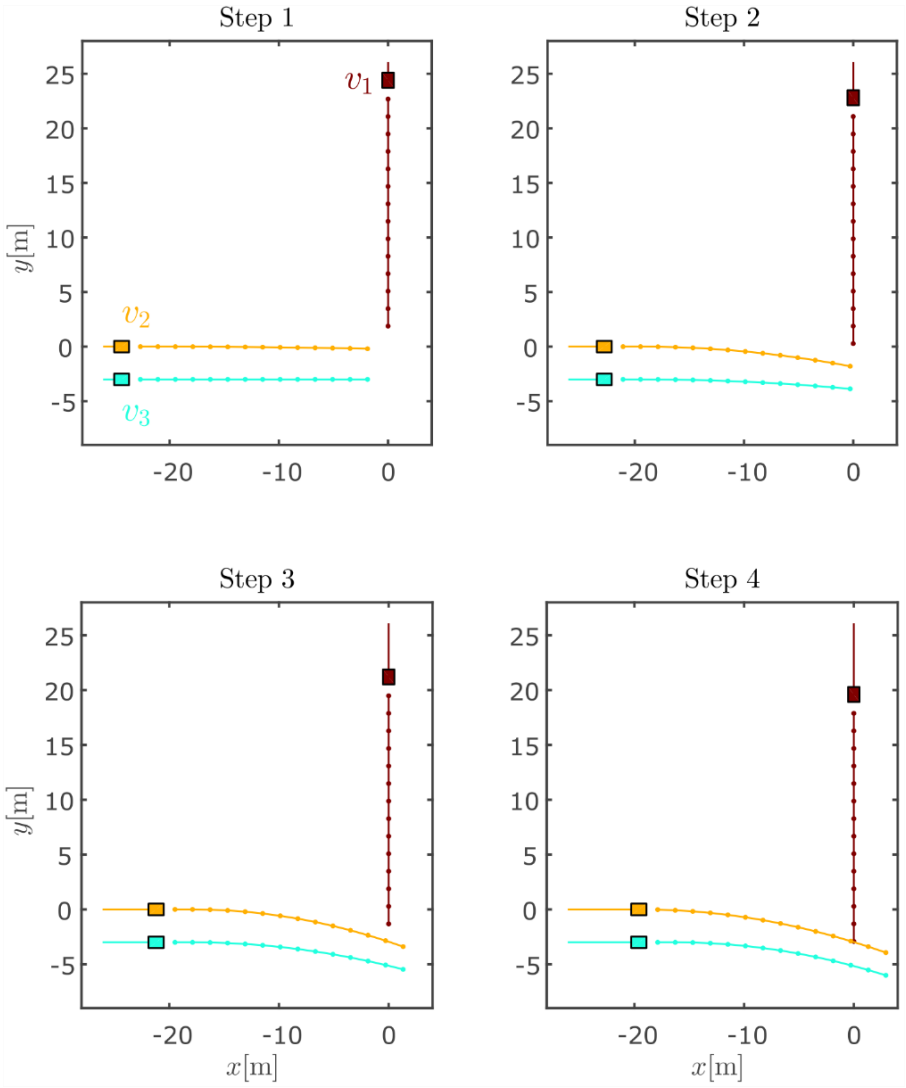


$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Rules of parallel and sequential computation:
  - Agents on the same level solve in parallel
  - If disturbances of agents on the first level are negligible, agents on the first and second level solve in parallel
  - Agents on level $3 \leq i \leq N_l$ solve sequentially after agents on level $i - 1$
  - The algorithm is executed repeatedly in the case of a change in the coupling graph

Net-MPC: priority-based non-cooperative distributed MPC

- Vehicles v_1 and v_2 solve in parallel
- Vehicle v_3 solves after v_1 and v_2
- Satisfaction of the prediction consistency property



Net-MPC: priority-based non-cooperative distributed MPC

Stability and feasibility discussion

- Main assumptions
  - A centralized MPC would generate a solution in each sample time that is NCS-stable, -feasible, and -optimal
  - Considering each agent as isolated from NCS, the solutions of PB-Non-Coop. DMPC are agent-stable, -feasible and -optimal
- Satisfaction of the prediction consistency property in any NCS even with time delays and time-variant coupling topology → Proof using mathematical induction
- Convergence after only one iteration of sequence

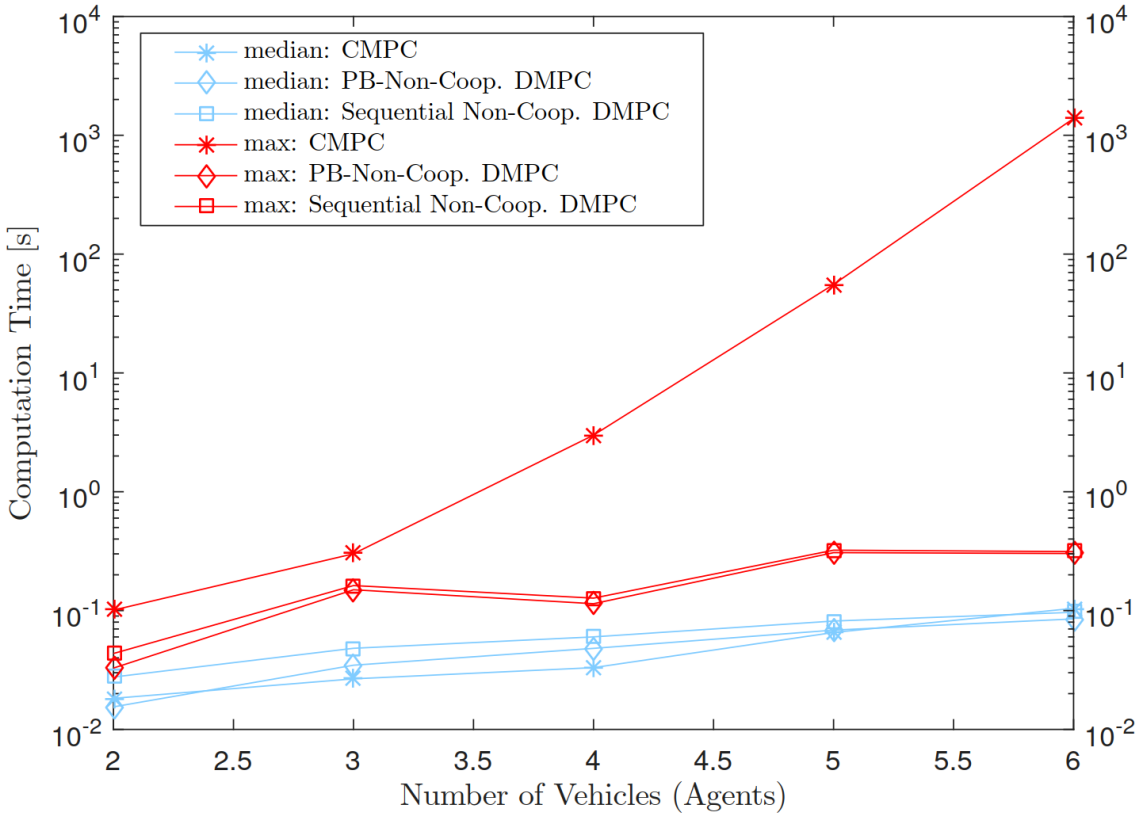
Net-MPC: comparison

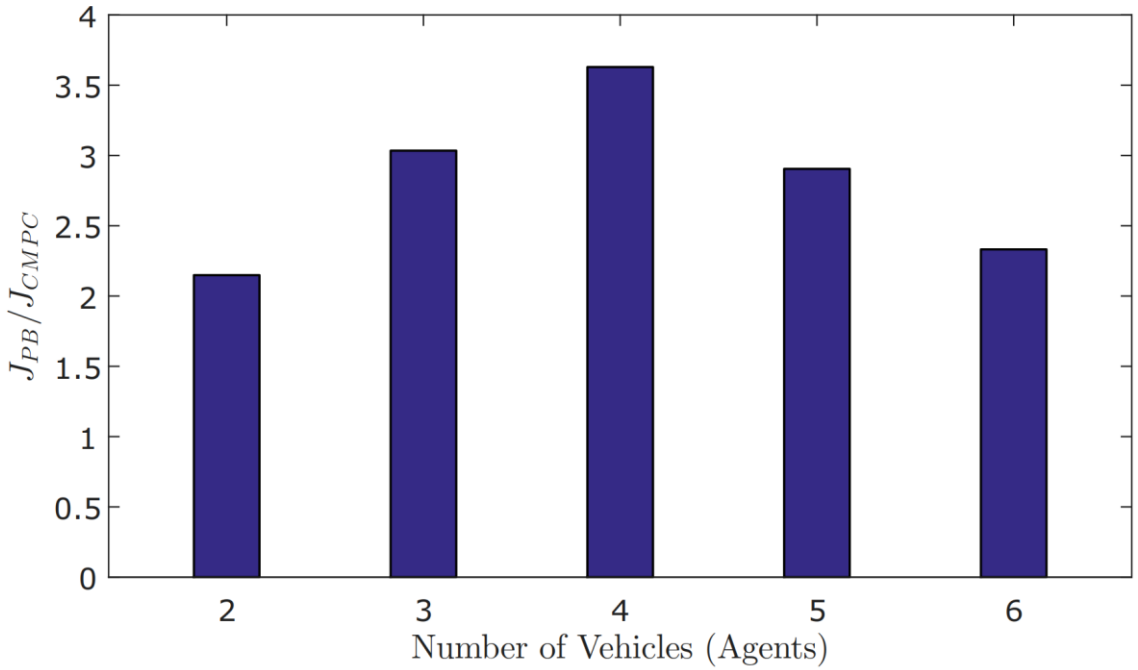
Optimization time of NCS in different Net-MPC strategies

Control Strategy	Optimization Time
CMPC	T_{ot}
Coop. DMPC	$T_{ot} = \max T_{ot}(v_i),\ \forall v_i \in \mathcal{V}$
PB-Non-Coop. DMPC	$T_{ot} = \max_{i \in level_{1,2}} T_{ot}(v_i) + \sum_{j=3}^{N_l} \max_{i \in level_j} T_{ot}(v_i)$
Sequential iterative DMPC	$T_{ot} = \sum_{1}^{N_{iter}} \sum_{i=1}^{N} T_{ot}(v_i)$

Net-MPC: comparison

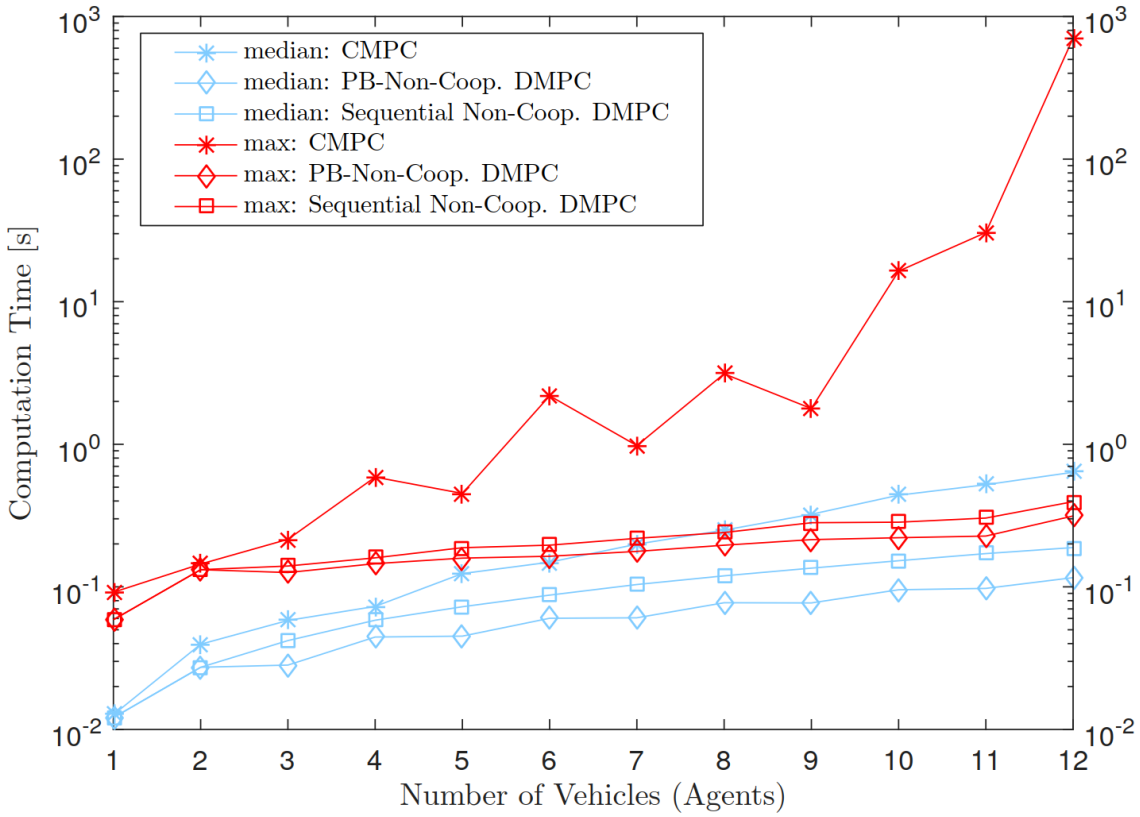
- Testing scenario N-circle, mean objective values normalized

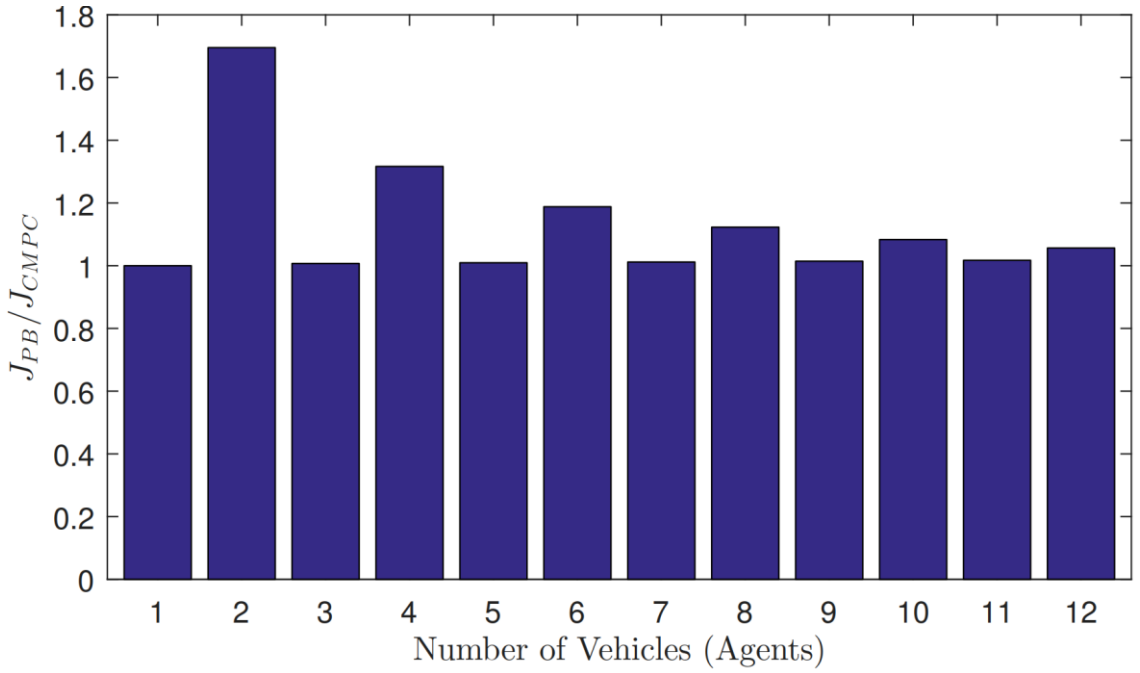




Net-MPC: comparison

- Testing scenario N-parallel, mean objective values normalized





Net-MPC: application to vehicle decision-making

- The application to vehicle decision-making combines high-level control methodologies and requirements of autonomous vehicles within a networked framework
- Networked vehicles are dynamically decoupled agents
- Couplings are functions of a subset of the states of a vehicle and a subset of the states of its neighbors
- (Active) agents are vehicles and passive agents are obstacles

- Net-MPC unifies collision detection, avoidance trajectory planning, and trajectory following in one optimization problem
- Efficient method to find a solution to the (in general non-convex) optimization problem of vehicle decision-making

Net-MPC: priority-based non-cooperative distributed MPC

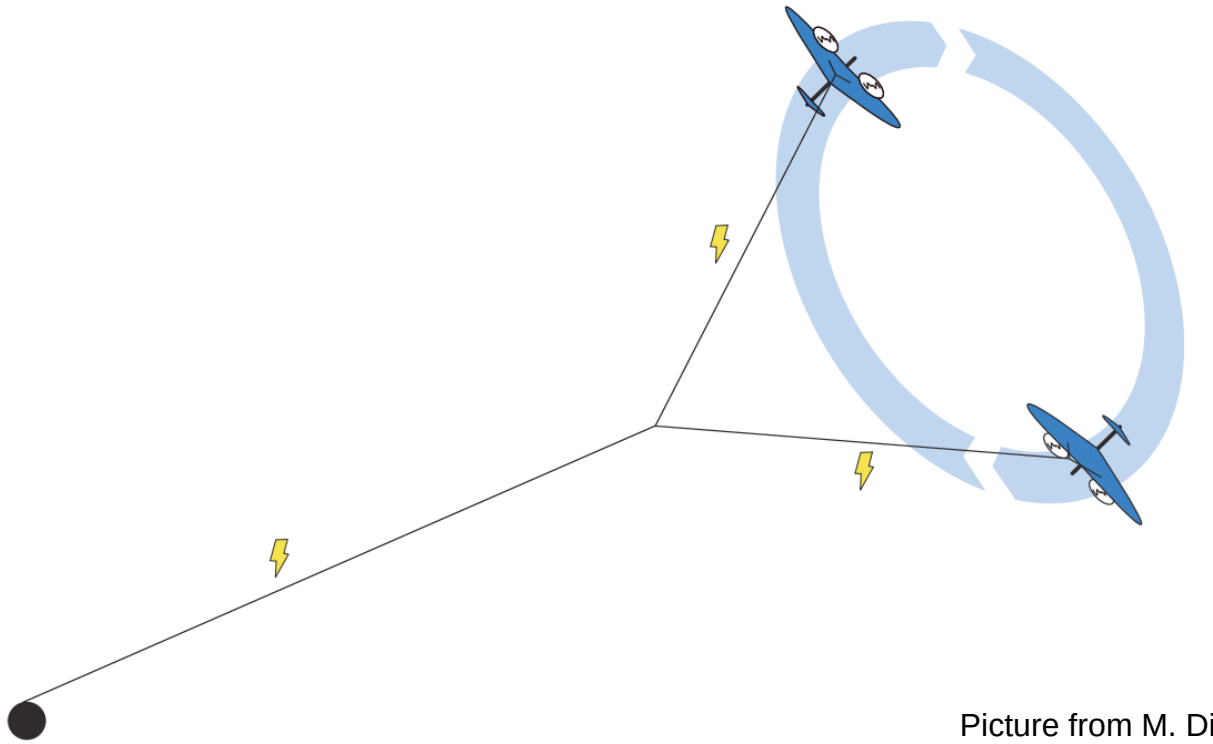
Vehicle examples

- Video of simulation results
  - https://youtu.be/zS3UBx09O6M
- Video of experimental results
  - https://youtu.be/X2syxG5GI6g
- Further simulation results
  - https://youtu.be/XGqI8FrjW6I
  - https://youtu.be/7sq3N8vwusA
  - https://youtu.be/kbooJFK52Fg

Net-MPC: priority-based non-cooperative distributed MPC

Application to aerial vehicles

- Airborne wind energy with a multiple wing system



Picture from M. Diehl, Airborne Wind Energy: Basic Concepts and Physical Foundations, 2013

Net-MPC: priority-based non-cooperative distributed MPC

Summary

- Developed for NCS consisting of dynamically decoupled agents
- Coupling in the objective function or in the constraints
- Consideration of time-variant coupling

- Satisfaction of prediction consistency property
- NCS-stable and -feasible
- Reduction of the computation time